



SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE HONOURS DEGREE IN STATISTICS AND OPERATIONS RESEARCH

LEVEL 4 SEMESTER 2

EXAMINATION QUESTION PAPER

MODULE CODE	HSOR 425
MODULE NARRATION	FUNDAMENTALS OF OPTIMIZATION
DATE	
DURATION	3 HOURS

INSTRUCTIONS TO CANDIDATES:

1. Candidates may attempt **FOUR** questions. All questions carry equal marks

GREAT ZIMBABWE UNIVERSITY

HSOR 425

SCHOOL OF NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

BSc HONOURS IN MATHEMATICS: PART 4 SEMESTER 2

EXAMINATION

HSOR 425: FUNDAMENTALS OF OPTIMIZATION

DATE:

Time : 3 hours

Candidates may attempt **FOUR** questions. All questions carry equal marks

A1. State and prove the

- (a) Second-order necessary conditions for a point to be a local minimizer. [12]
 (b) Karush-Kuhn-Tucker (KKT) Theorem. [13]

A2. (a) If $\{x^{(k)}\}_{k=0}^{\infty}$ is a steepest descent sequence for a given function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, prove that for each k the vector $x^{(k+1)} - x^{(k)}$ is orthogonal to the vector $x^{(k+2)} - x^{(k+1)}$. [9]
 (b) Prove that we have $x^{(k)} \rightarrow x^*$ for any $x^{(0)}$ in the steepest descent algorithm. [7]
 (c) State and prove the Lagrange's Theorem. [9]

A3. (a) Use the Golden Section search to find the value of x that minimizes

$$f(x) = 4x^2 + 2\cos x$$

in the range $[1, 2]$. Locate this value of x to within a range of 0.2. [15]

- (b) Use the the Fibonacci method to solve the problem in (a) with $\epsilon = 0.05$. [10]

A4. (a) Given the optimal primal basis \mathbf{B} and its associated objective coefficient vector \mathbf{C}_B , show that the optimal solution of the dual problem is

$$\mathbf{Y} = \mathbf{C}_B \mathbf{B}^{-1}.$$

[9]

- (b) Consider the following problem:

$$f(X) = x_1^2 + 3x_2^2 + 5x_1x_3^2$$

subject to

$$g_1(X) = x_1x_3 + 2x_2 + x_2^2 - 11 = 0,$$

$$g_2(X) = x_1^2 + 2x_1x_2 + x_3^2 - 14 = 0.$$

Given the feasible point $X^0 = (1, 2, 3)$, determine the variation in $f(= \partial_c f)$ in the feasible neighbourhood of X^0 . [10]

- (c) Solve the problem:

Minimize

$$f(X) = x_1^2 + x_2^2 + x_3^2$$

subject to

$$g_1(X) = x_1 + x_2 + 3x_3 - 2 = 0$$

$$g_2(X) = 5x_1 + 2x_2 + x_3 - 5 = 0.$$

[6]

- A5.** (a) If $\{x^{(k)}\}_{k=0}^{\infty}$ is a steepest descent sequence for a given function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, prove that for each k the vector $x^{(k+1)} - x^{(k)}$ is orthogonal to the vector $x^{(k+2)} - x^{(k+1)}$. [10]
- (b) For a function $f_o : \mathbb{R}^n \rightarrow \mathbb{R}$ of class C^2 , consider the problem of minimizing f_o over all $x \in \mathbb{R}^n$. State and prove the
- (i) necessary condition. [6]
 - (ii) sufficient condition. [9]

END OF QUESTION PAPER