GARY MAGADZIRE SCHOOL OF AGRICULTURE AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE HONOURS DEGREE IN STATISTICS AND OPERATIONS RESEARCH

LEVEL 2 SEMESTER 2/ 2 SEMESTER 1

EXAMINATION QUESTION PAPER

MODULE CODE	**HSOR 222**
MODULE NARRATION	**STOCHASTIC PROCESS 1**
DATE	
DURATION	**3 HOURS**

INSTRUCTIONS TO CANDIDATES:

1. Candidates may attempt **ALL** questions in Section A and ANY TWO questions from Section B

Great Zimbabwe University
BSc Statistics and Operations Research
Examination: Stochastic Processes 1
Time: 3 Hours
Total Marks: 100

Answer all questions in Section A and two questions from Section B.

Section A: (40 marks)

Question 1

(20 marks)

(a) Define a Poisson process. Show that the time between successive events in a Poisson process follows an exponential distribution. (10 marks)

(b) A manufacturing plant produces items where defects occur randomly in time according to a Poisson process with a rate of $\lambda = 3$ defects per hour. Find the probability that exactly 2 defects occur in a 1-hour interval. (5 marks)

(c) Suppose the number of machine failures in a day at a factory follows a Poisson process with rate $\lambda = 5$ failures per day. Calculate the probability that the factory experiences no failures in a given day. (5 marks)

Question 2

(20 marks)

(a) Define a Markov process and give an example of a continuous-time Markov chain that can be used to model inventory levels in a warehouse. (10 marks)

(b) A company uses a Markov process to model employee retention and promotion. The company has two states: employed (E) and promoted (P). The transition matrix is given as:

$$P = \begin{pmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{pmatrix}$$

Determine the long-term probabilities of being in each state. (10 marks)

Section B: (30 marks each)
Answer any two questions.
Question 3
(30 marks)

(a) Define a renewal process. Explain how renewal processes can be used in industrial settings to model the lifetime and replacement of machinery. (10 marks)

(b) In an assembly line, machines have an average lifetime of 10 hours, and replacement time is negligible. Assume the machine lifetimes follow an exponential distribution. Find the expected number of replacements needed in a 24-hour production shift. (10 marks)

(c) Explain how renewal theory can be used to determine the optimal time to replace machinery in order to minimize downtime. (10 marks)

Question 4
(30 marks)

(a) Define a birth-death process. Provide an example of a birth-death process in the context of inventory management. (10 marks)

(b) A company's product demand follows a birth-death process where the birth rate is 2 products per hour, and the death rate is 1 product per hour. Find the probability that there are exactly 3 products in demand at a given time. (10 marks)

(c) Discuss the application of birth-death processes in modeling customer arrivals and service completion in a call center. (10 marks)

Question 5
(30 marks)

(a) Explain the concept of a random walk and how it can be used to model stock price movements. (10 marks)

(b) A company's stock price follows a simple random walk. At each step, the price increases by \$1 with probability 0.4 or decreases by \$1 with probability 0.6. If the current price is \$50, what is the probability that the stock will hit \$55 before it hits \$45? (10 marks)

(c) Discuss the practical limitations of using random walk models in financial markets. (10 marks)

Question 6
(30 marks)

(a) Define a queuing system and explain how the $M/M/1$ queue can be used to model customer arrivals and service in a bank. (10 marks)

(b) In an $M/M/1$ queuing system, customers arrive at a bank according to a Poisson process with rate $\lambda = 5$ customers per hour, and the service times follow an exponential distribution with rate $\mu = 7$ customers per hour. Compute the average number of customers in the system. (10 marks)

(c) Explain how queuing models can help improve efficiency in manufacturing processes. (10 marks)

End of Examination