ROBERT MUGABE SCHOOL OF HERITAGE & EDUCATION

DEPARTMENT OF SCIENCE AND TECHNICAL EDUCATION

BACHELOR OF EDUCATION SECONDARY HONOURS PRE-SERVICE/IN-SERVICE DEGREE

PART 1 SEMESTER 2/PART 1 SEMESTER1

EXAMINATION QUESTION PAPER

MODULE CODE : TDSM 121 /CMTS112

NARRATION : LINEAR ALGEBRA /LINEAR MATHEMATICS

DATE : 2025

TIME : 3HOURS

MAIN PAPER

INSTRUCTIONS

Answer any FIVE questions in all. Each question carries 20 marks.

1. (a) Define and give examples of the following concepts in matrix algebra:

(i) a symmetric matrix $B=(b_{ij})_{m \times m}$ [3]

(ii) a lower triangular matrix $A=(a_{ij})_{n \times n}$ [3]

(iii) transpose of a matrix $C = (c_{ij})_{n \times n}$ [3]

(b) Let W be the space generated by the polynomials

$v1 = t^3 - 2t^2 + 3$

$v2 = 2t^3 + 3t^2 + t - 4$

$v3 = 3t^3 + 8t^2 - 3t - 5$

Find a basis and the dimension of W [5, 2]

(c) Given the vectors u = (1, 2, -1) and v = (6, 4, 2) in $\mathbf{R}^3$, show that w = (9, 2, 7) is a linear combination of u and v. [4]

2. (a) Define a linear mapping. [3]

(b) Determine whether or not the following mappings are linear.

(i) $F: R^2 \to R^2$ defined by $F(x, y) = (x + y, x)$. [4]

(ii) $F: R^2 \to R^3$ defined by $F(x, y) = (x + 1, 2y, x + y)$. [4]

(c) Let $T: V \to W$ be a linear mapping where V and W are vector spaces.

Define

(i)Ker (T). [2]

(ii) Im (T). [2]

(e) Prove that the range of T is a subspace of W. [5]

3 (a) Use Gaussian elimination to solve the following system of linear equations:

$$x + 4y + 2z = 0$$

$$3x + 5y + 3z = -1$$

$$2y + z = 3$$ [8]

(b) Determine the values of k so that the following system in unknowns x, y and z has

(a) no solution,
(b) more than one solution,
(c) a unique solution.

$$x + y + kz = 1,$$
$$x + ky + z = 1,$$
$$kx + y + z = 1. \qquad [12]$$

4. (a) Consider the matrix: $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$

(i) Find the characteristic polynomial. [4]

(ii) Find all eigenvalues of A. [6]

(iii) Find the eigenvector for each eigenvalue of A. [6]

(b) Given that $A = \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{pmatrix}$. Find $|A|$. [4]

5. (a)(i) Use elementary row operations to find the inverse of matrix A below.

$$A = \begin{pmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{pmatrix}. \qquad [8]$$

(ii) Find AA^T. [4]

(b) (i) State Cayley-Hamilton's Theorem. [3]

(ii) Verify Cayley-Hamilton's Theorem for the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$. [5]

6. Let V be the vector space of polynomials of degree ≤ 3 over R. Determine whether u, v, w $\in$ V are dependent or independent where:

$$u = t^3 + 4t^2 - 2t + 3$$

$$v = t^3 + 6t^2 - t + 4$$

$$w = 3t^3 - 8t^2 - 8t + 7 \qquad [7]$$

7. Let T (x,y) = (2y, 3x-y) be a linear operator on $\mathbf{R}^2$.

(a) Find the matrix representation of T

(i) relative to the usual basis $\{e_1 = (1, 0), e_2 = (0, 1)\}$, and [3]

(ii) relative to the basis $\{f_1 = (1, 3), f_2 = (2, 5)\}$. [5]

(b) (i) Find the transition matrix P from the basis

$\{e_1 = (1, 0), e_2 = (0, 1)\}$, to the basis

$\{f_1 = (1, 3), f_2 = (2, 5)\}$. [3]

(ii) Find the transition matrix Q from the basis

$\{f_1 = (1, 3), f_2 = (2, 5)\}$ to the basis

$\{e_1 = (1, 0), e_2 = (0, 1)\}$. [4]

(iii) Verify that $Q = P^{-1}$. [2]

(iv) Show that $[v]_f = P^{-1}[v]_e$ for any vector $v \in \mathbf{R}^2$. [3]

END OF EXAMINATION