HERBERT CHITEPO SCHOOL OF LAW AND BUSINESS SCIENCES

DEPARTMENT OF ECONOMICS AND FINANCE

BACHELOR OF COMMERCE DEGREE

LEVEL 1 SEMESTER 1

EXAMINATION QUESTION PAPER

MODULE CODE	**HEC 116**
MODULE NARRATION	**STATISTICS FOR ECONOMICS 1**
DATE	**2024**
DURATION	**3 HOURS**

INSTRUCTIONS TO CANDIDATES:

1. Answer 4 questions
2. Start each question on a fresh page
3. Silent self-powered non-programmable calculators can be used
4. All questions carry equal marks

Question 1

(a) A university administrator requested a breakdown of travel expenses for faculty to attend various professional meetings. It was found that 31% of the travel expenses was spent for transportation costs, 25% was spent for lodging, 17% was spent for food, and 20% was spent for conference registration fees; the remainder was spent for miscellaneous costs.

(i) Construct a pie chart. [5 marks]
(ii) Construct a bar chart. [5 marks]

(b) Differentiate the following terms giving practical examples:

(i) Ordinal and nominal data [5 marks]
(ii) Interval and ratio data [5 marks]
(iii) Discrete and continuous data [5 marks]

[Total 25 marks]

Question 2

(a) The setting time for ceramic tile glue is an important quality feature of the product. A manufacturer of ceramic tile glue tested a sample of nine batches from a large consignment and recorded the setting times (in minutes) of each batch:

27	22	31	18	20	25	21	28	24

(i) Find the mean and standard deviation of setting times of the ceramic tile glue. [5 marks]
(ii) How consistent are the setting times across the different batches? Compute the coefficient of variation as a consistency index measure. [5 marks]
(iii) If the consistency index must be less than 10% for the consignment to be passed by the quality controller, will this consignment be approved for dispatch? Explain your answer. [3 marks]

(b) The human resources department of a company recorded the number of days absent of 23 employees in the technical department over the past nine months:

5	4	8	17	10	9	30	5	6	15	10	9
2	16	15	18	4	12	6	6	15	10	5	

(i) Find the mean, median and modal number of days absent over this nine-month period. Interpret each central location measure. [4 marks]

(ii) Compute the first quartile and the third quartile of the number of days absent. Interpret these quartile values for the human resources manager. [4 marks]

(iii) The company's policy is to keep its absenteeism level to within an average of one day per employee per month. Based on the findings in (a), is the company successful in managing its absenteeism level? Explain [4 marks]

[Total 25 marks]

Question 3

The following table shows the 300 employees of a glass manufacturing company, cross-classified on the basis of age and department:

Age	Department			Total
	Production	Sales	Administration	
< 30	60	25	18	103
30-50	70	29	25	124
>50	30	8	35	73
Total	160	62	78	300

(a) An employee is selected at random from this company. Calculate the probability that the employee is:

(i) Under 30 years of age; [2 marks]

(ii) a production worker; [2 marks]

(iii) a sales person and between 30 and 50 years of age; [2 marks]

(iv) over 50, given that he or she is in administration; [2 marks]

(v) A production worker or under 30 years, or both. [2 marks]

(b) A child helpline receives calls that can be described by the Poisson process. The average rate at which calls come in is three calls per minute.

(i) Find the probability that the helpline will receive exactly five calls in a given minute [3 marks]

(ii) What is the likelihood that the telephone will receive four or more calls in a given minute?

[4 marks]

(iii) What chance is there that no calls will be received in a given minute? [3 marks]

[Total 25 Marks]

QUESTION 4

A web-based travel agency uses its website to market its travel products (holiday packages). The agency receives an average of five web-based enquiries per day for its different travel products.

(a) What is the probability that, on a given day, the agency will receive only three web-based enquiries for its travel products? [6 marks]

(b) What is the probability that, on a given day, the travel agency will receive at most two web-based enquiries for travel packages? [6 marks]

(c) What is the probability that the travel agency will receive more than four web-based enquiries for travel packages on a given day? [6 marks]

(d) What is the probability that the travel agency will receive more than four web-based enquiries for travel packages in any two-day period? [7 marks]

[Total 25 Marks]

QUESTION 5

(a) Find the following probabilities using the standard normal z-tables. Give a sketch with the appropriate area shaded in.

(i) $P(0 < z < 1.83)$ [2 marks]

(ii) $P(z > -0.48)$ [2 marks]

(iii) $P(-2.25 < z < 0)$ [2 marks]

(iv) $P(1.22 < z)$ [2 marks]

(v) $P(-2.08 < z < 0.63)$ [2 marks]

(b) A machine filling 18-gram containers of a hair dye is set so that the average fill is 18.2 grams with a variance of 0.49 grams. Assume that the filling of containers by this machine is normally distributed.

(i) What percentage of the containers is not likely to meet the producer's specification of at least 18 grams per container? [8 marks]

(ii) What is the minimum mass of the heaviest 15% of containers? [7 marks]

[Total 25 Marks]

END OF EXAMINATION

APPENDIX 4: LIST OF KEY FORMULAE

MEASURES OF CENTRAL LOCATION

Arithmetic mean *Ungrouped data*

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad 3.1$$

Grouped data

$$\bar{x} = \frac{\sum_{i=1}^{m} f_i x_i}{n}$$

Mode *Grouped data*

$$M_o = O_{mo} + \frac{c(f_m - f_{m-1})}{2f_m - f_{m-1} - f_{m+1}} \qquad 3.3$$

Median *Grouped data*

$$M_e = O_{me} + \frac{c\left[\frac{n}{2} - f(<)\right]}{f_{me}} \qquad 3.2$$

Lower quartile *Grouped data*

$$Q_1 = O_{Q1} + \frac{c\left(\frac{n}{4} - f(<)\right)}{f_{Q1}} \qquad 3.7$$

Upper quartile *Grouped data*

$$Q_3 = O_{Q3} + \frac{c\left(\frac{3n}{4} - f(<)\right)}{f_{Q3}} \qquad 3.8$$

Geometric mean *Ungrouped data*

$$GM = \sqrt[n]{x_1 . x_2 . x_3 \ldots x_n} \qquad 3.4$$

Weighted arithmetic mean *Grouped data*

$$\text{weighted } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} \qquad 3.5$$

MEASURES OF DISPERSION AND SKEWNESS

Range Range = Maximum value – Minimum value

$$= x_{max} - x_{min} \qquad 3.9$$

Variance *Mathematical – ungrouped data*

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{(n-1)} \qquad 3.10$$

Computational – ungrouped data

$$s^2 = \frac{\Sigma x_i^2 - n\bar{x}^2}{(n-1)} \qquad 3.11$$

Standard deviation

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} \qquad 3.12$$

Coefficient of variation

$$CV = \tfrac{s}{\bar{x}} \times 100\% \qquad 3.13$$

Pearson's coefficient of skewness

$$Sk_p = \frac{n\Sigma(x_i - \bar{x})^3}{(n-1)(n-2)s^3} \qquad 3.14$$

$$Sk_p = \frac{3\ (\text{Mean} - \text{Median})}{\text{Standard deviation}} \quad \text{(approximation)} \qquad 3.15$$

PROBABILITY CONCEPTS

Probability

$$P(A) = \tfrac{r}{n} \qquad 4.1$$

Conditional probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \qquad 4.2$$

Addition rule *Non-mutually exclusive events*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad 4.3$$

Mutually exclusive events

$$P(A \cup B) = P(A) + P(B) \qquad 4.4$$

Multiplication rule	*Statistically dependent events* $P(A \cap B) = P(A \mid B) \times P(B)$	**4.5**
	Statistically independent events $P(A \cap B) = P(A) \times P(B)$	**4.6**
Independence test	$P(A \mid B) = P(A)$	**4.7**
Bayes' Theorem	$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$	**4.8**
***n*! = *n* factorial**	$n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 3 \times 2 \times 1$	**4.9**
Permutations	${}_nP_r = \frac{n!}{(n-r)!}$	**4.11**
Combinations	${}_nC_r = \frac{n!}{r!\,(n-r)!}$	**4.12**

PROBABILITY DISTRIBUTIONS

Binomial distribution	$P(x) = {}_nC_x p^x (1-p)^{(n-x)}$ for $x = 0, 1, 2, 3, \ldots, n$	**5.1**
Binomial descriptive measures	Mean $\mu = np$ Standard deviation $\sigma = \sqrt{np(1-p)}$	**5.2**
Poisson distribution	$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ for $x = 0, 1, 2, 3 \ldots$	**5.3**
Poisson descriptive measures	Mean $\mu = \lambda$ Standard deviation $\sigma = \sqrt{\lambda}$	**5.4**
Standard normal probability	$z = \frac{x - \mu}{\sigma}$	**5.6**

CONFIDENCE INTERVALS

Single mean	*n large; variance known* $\bar{x} - z\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z\frac{\sigma}{\sqrt{n}}$ (lower limit) (upper limit)	**7.2**

n small; variance unknown

$$\bar{x} - t_{(n-1)}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{(n-1)}\frac{s}{\sqrt{n}} \qquad 7.3$$

(lower limit) (upper limit)

Single proportion

$$p - z\sqrt{\frac{p(1-p)}{n}} \le \pi \le p + z\sqrt{\frac{p(1-p)}{n}} \qquad 7.5$$

(lower limit) (upper limit)

Sample Size *– mean*

$$n = \frac{z^2\sigma^2}{e^2} \qquad 7.6$$

Sample Size *– proportions*

$$n = z^2\frac{p(1-p)}{e^2} \qquad 7.7$$

HYPOTHESES TESTS

Single mean

Variance known

$$z\text{-}stat = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \qquad 8.1$$

Variance unknown; n small

$$t\text{-}stat = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \qquad 8.2$$

Single proportion

$$z\text{-}stat = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \qquad 8.3$$

Single variance

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \qquad 8.4$$

Difference between two means

Variances known

$$z\text{-}stat = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \qquad 9.1$$

Pooled-variances t-test

$$t\text{-}stat = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\text{where } s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \qquad 9.2$$

Unequal-variances t-test

$$t\text{-}stat = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$\text{with } df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{(n_1 - 1)} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{(n_2 - 1)}}$$ 9.9

Paired t-test $$t\text{-}stat = \frac{\bar{x}_d - \mu_d}{\frac{s_d}{\sqrt{n}}}$$ 9.5

where $\mu_d = (\mu_1 - \mu_2)$

and $s_d = \sqrt{\frac{\Sigma(x_d - \bar{x}_d)^2}{n-1}}$

Differences between two proportions $$z\text{-}stat = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\hat{\pi}(1 - \hat{\pi})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\hat{\pi} = \frac{x_1 + x_2}{n_1 + n_2}$ $p_1 = \frac{x_1}{n_1}$ $p_2 = \frac{x_2}{n_2}$ 9.8

Equality of variances $$F\text{-}stat = \frac{sample\ variance_1}{sample\ variance_2} = \frac{s_1^2}{s_2^2}$$ 9.10

Chi-Square $$\chi^2\text{-}stat = \Sigma\frac{(f_o - f_e)^2}{f_e} \text{ with } df = (r-1)(c-1)$$ 10.1

ANALYSIS OF VARIANCE – ONE FACTOR

Overall mean $$\bar{\bar{x}} = \frac{\Sigma\Sigma x_{ij}}{N}$$ 11.2

SSTotal $$= \sum_i \sum_j (x_{ij} - \bar{\bar{x}})^2$$ 11.3

SST $$= \sum_j^k n_j(\bar{x}_j - \bar{\bar{x}})^2$$ 11.4

SSE $$= \sum_j \sum_i (x_{ij} - \bar{x}_j)^2$$ 11.5

ANALYSIS OF VARIANCE – TWO FACTOR

SSTotal $= \sum_a \sum_b \sum_k (x_{ijk} - \bar{x})^2$ 11.11

SSA $= bk \sum_j^a (\bar{x}_j[A] - \bar{\bar{x}})^2$ 11.12

SSB $= ak \sum_i^b (\bar{x}_i[B] - \bar{\bar{x}})^2$ 11.13

SS(AB) $=$ SSTotal – SSA – SSB – SSE 11.14

SSE $= \sum^a \sum^b \sum^k (x_{ijk} - \bar{x}_{ij}[AB])^2$ 11.15

REGRESSION AND CORRELATION

Formula $\hat{y} = b_0 + b_1 x$ 12.1

Coefficients $b_1 = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$ 12.2

$b_0 = \frac{\Sigma y - b_1 \Sigma x}{n}$ 12.3

Pearson's correlation coefficient $r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2] \times [n\Sigma y^2 - (\Sigma y)^2]}}$ 12.4

t-stat $= r\sqrt{\frac{(n-2)}{1-r^2}}$ 12.8

$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + \dots\dots\dots + b_p x_p$ 13.1

$R^2 = \frac{\text{SS(Regression)}}{\text{SS(Total)}}$ 13.2

$S_e = \sqrt{\frac{\Sigma(y_i - \hat{y}_i)^2}{n-p-1}}$ 13.3

$F\text{-stat} = \frac{\text{Variation Explained by Regression}}{\text{Unexplained Variation}} = \frac{\text{MS Regression}}{\text{MS Error}}$ 13.4

$$t\text{-}stat = \frac{(b_i - \beta_i)}{S_{b_i}} \quad 13.5$$

$$b_i - (t\text{-}crit) \times std\ error\ (b_i) \leq \beta_i \leq b_i + (t\text{-}crit) \times std\ error\ (b_i) \quad 13.6$$

$$\hat{y} \pm t\left(\tfrac{\alpha}{2}, n-p-1\right)\left(\frac{\text{standard error}}{\sqrt{n}}\right) \quad 13.7$$

INDEX NUMBERS

Price relative

$$\text{Price relative} = \frac{p_1}{p_0} \times 100\% \quad 14.2$$

Laspeyres price index

Weighted aggregates method

$$\text{Laspeyres price index} = \frac{\Sigma(p_1 \times q_0)}{\Sigma(p_0 \times q_0)} \times 100\% \quad 14.5$$

Laspeyres price index

Weighted average of relatives method

$$\text{Laspeyres price index} = \frac{\Sigma\left[\left(\frac{p_1}{p_0}\right) \times 100 \times (p_0 \times q_0)\right]}{\Sigma(p_0 \times q_0)} \quad 14.9$$

Paasche price index

Weighted aggregates method

$$= \frac{\Sigma(p_1 \times q_1)}{\Sigma(p_0 \times q_1)} \times 100\% \quad 14.8$$

Paasche price index

Weighted average of relatives method

$$= \frac{\Sigma\left[\left(\frac{p_1}{p_0}\right) \times 100 \times (p_0 \times q_1)\right]}{\Sigma(p_0 \times q_1)} \quad 14.10$$

Quantity relative

$$\text{Quantity relative} = \frac{q_1}{q_0} \times 100\% \quad 14.11$$

Laspeyres quantity index

Weighted aggregates method

$$\text{Laspeyres quantity index} = \frac{\Sigma(p_0 \times q_1)}{\Sigma(p_0 \times q_0)} \times 100\% \quad 14.12$$

Laspeyres quantity index *Weighted average of relatives method*

$$\text{Laspeyres quantity index} = \frac{\Sigma\left[\left(\frac{q_1}{q_0}\right) \times 100 \times (p_0 \times q_0)\right]}{\Sigma(p_0 \times q_0)} \qquad 14.14$$

Paasche quantity index *Weighted aggregates method*

$$= \frac{\Sigma(p_1 \times q_1)}{\Sigma(p_1 \times q_0)} \times 100\% \qquad 14.13$$

Paasche quantity index *Weighted average of relatives method*

$$= \frac{\Sigma\left[\left(\frac{q_1}{q_0}\right) \times 100 \times (p_1 \times q_0)\right]}{\Sigma(p_1 \times q_0)} \qquad 14.15$$

Link relatives *Price*

$$= \frac{p_i}{p_{i-1}} \times 100\% \qquad 14.17$$

Quantity

$$= \frac{q_i}{q_{i-1}} \times 100\% \qquad 14.18$$

Composite

$$= \frac{\text{Basket value}_i}{\text{Basket value}_{i-1}} \times 100\% \qquad 14.19$$

or

$$= \frac{\text{Composite index}_i}{\text{Composite index}_{i-1}} \times 100\%$$

TIME SERIES ANALYSIS

Regression trend coefficients

$$b_1 = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \qquad 12.2$$

$$b_0 = \frac{\Sigma y - b_1\Sigma x}{n} \quad \text{where } x = 1, 2, 3, 4 \ldots n \qquad 12.3$$

De-seasonalised *y*

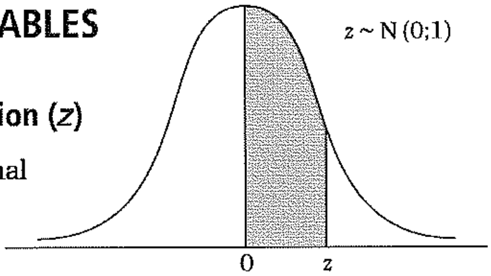
$$= \frac{\text{Actual } y}{\text{Seasonal index}} \times 100 \qquad 15.5$$

APPENDIX 1: LIST OF STATISTICAL TABLES

TABLE 1 The standard normal distribution (*z*)

This table gives the area under the standard normal curve between 0 and *z*

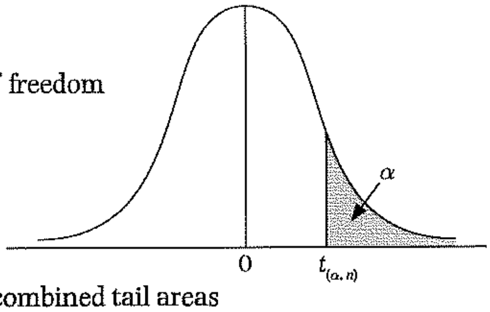
P[0 < Z < z]



In *Excel* (2013): use NORM.S.DIST(z, cumulative = True) to find P(– ∞ < Z < z)

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2703	0.2734	0.2764	0.2793	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3557	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49897	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49991	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

TABLE 2 The *t* distribution



This table gives the value of $t_{(\alpha, n)}$ with n degrees of freedom

[shaded area] $= P[t \geq t_{(\alpha, n)}]$

In *Excel* (2013) use:

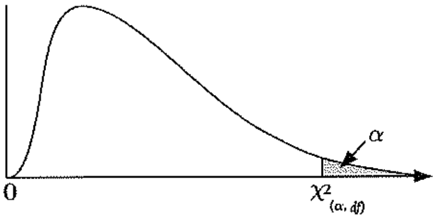
- T.INV(α, *df*) for a one-sided lower limit
- T.INV($1 - \alpha$, *df*) for a one-sided upper limit
- T.INV.2T(α, *df*) for two-sided limits where α = combined tail areas

α / *df*	0.100	0.050	0.025	0.010	0.005	0.0025
1	3.078	6.314	12.706	31.821	63.657	127.322
2	1.886	2.920	4.303	6.965	9.925	14.089
3	1.638	2.353	3.182	4.541	5.841	7.453
4	1.533	2.132	2.776	3.747	4.604	5.598
5	1.476	2.015	2.571	3.365	4.032	4.773
6	1.440	1.943	2.447	3.143	3.707	4.317
7	1.415	1.895	2.365	2.998	3.499	4.029
8	1.397	1.860	2.306	2.896	3.355	3.833
9	1.383	1.833	2.262	2.821	3.250	3.690
10	1.372	1.812	2.228	2.764	3.169	3.581
11	1.363	1.796	2.201	2.718	3.106	3.497
12	1.356	1.782	2.179	2.681	3.055	3.428
13	1.350	1.771	2.160	2.650	3.012	3.372
14	1.345	1.761	2.145	2.624	2.977	3.326
15	1.341	1.753	2.131	2.602	2.947	3.286
16	1.337	1.746	2.120	2.583	2.921	3.252
17	1.333	1.740	2.110	2.567	2.898	3.222
18	1.330	1.734	2.101	2.552	2.878	3.197
19	1.328	1.729	2.093	2.539	2.861	3.174
20	1.325	1.725	2.086	2.528	2.845	3.153
21	1.323	1.721	2.080	2.518	2.831	3.135
22	1.321	1.717	2.074	2.508	2.819	3.119
23	1.319	1.714	2.069	2.500	2.807	3.104
24	1.318	1.711	2.064	2.492	2.797	3.091
25	1.316	1.708	2.060	2.485	2.787	3.078
26	1.315	1.706	2.056	2.479	2.779	3.067
27	1.314	1.703	2.052	2.473	2.771	3.057
28	1.313	1.701	2.048	2.467	2.763	3.047
29	1.311	1.699	2.045	2.462	2.756	3.038
30	1.310	1.697	2.042	2.457	2.750	3.030
31	1.309	1.696	2.040	2.453	2.744	3.022
32	1.309	1.694	2.037	2.449	2.738	3.015
33	1.308	1.692	2.035	2.445	2.733	3.008
34	1.307	1.691	2.032	2.441	2.728	3.002
35	1.306	1.690	2.030	2.438	2.724	2.996
36	1.306	1.688	2.028	2.434	2.719	2.990
37	1.305	1.687	2.026	2.431	2.715	2.985
38	1.304	1.686	2.024	2.429	2.712	2.980
39	1.304	1.685	2.023	2.426	2.708	2.976
40	1.303	1.684	2.021	2.423	2.704	2.971
45	1.301	1.679	2.014	2.412	2.690	2.952
50	1.299	1.676	2.009	2.403	2.678	2.937
60	1.296	1.671	2.000	2.390	2.660	2.915
70	1.294	1.667	1.994	2.381	2.648	2.899
80	1.292	1.664	1.990	2.374	2.639	2.887
90	1.291	1.662	1.987	2.369	2.632	2.878
100	1.290	1.660	1.984	2.364	2.626	2.871
120	1.289	1.658	1.980	2.358	2.617	2.860
140	1.288	1.656	1.977	2.353	2.611	2.852
160	1.287	1.654	1.975	2.350	2.607	2.847
180	1.286	1.653	1.973	2.347	2.603	2.842
200	1.286	1.653	1.972	2.345	2.601	2.839
∞	1.282	1.645	1.960	2.327	2.576	2.807

TABLE 3 The Chi-square distribution (χ^2)

The entries in this table are critical χ^2 limits where α is the area to the right of the critical limit.

In *Excel* (2013): CHISQ.INV.RT(α, *df*)



α df	0.005	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	0.995
1	7.879	6.635	5.024	3.841	2.706	0.016	0.004	0.001	0.000	0.000
2	10.597	9.210	7.378	5.991	4.605	0.211	0.103	0.051	0.020	0.010
3	12.838	11.345	9.348	7.815	6.251	0.584	0.352	0.216	0.115	0.072
4	14.860	13.277	11.143	9.488	7.779	1.064	0.711	0.484	0.297	0.207
5	16.750	15.086	12.833	11.070	9.236	1.610	1.145	0.831	0.554	0.412
6	18.548	16.812	14.449	12.592	10.645	2.204	1.635	1.237	0.872	0.676
7	20.278	18.475	16.013	14.067	12.017	2.833	2.167	1.690	1.239	0.989
8	21.955	20.090	17.535	15.507	13.362	3.490	2.733	2.180	1.646	1.344
9	23.589	21.666	19.023	16.919	14.684	4.168	3.325	2.700	2.088	1.735
10	25.188	23.209	20.483	18.307	15.987	4.865	3.940	3.247	2.558	2.156
11	26.757	24.725	21.920	19.675	17.275	5.578	4.575	3.816	3.053	2.603
12	28.300	26.217	23.337	21.026	18.549	6.304	5.226	4.404	3.571	3.074
13	29.819	27.688	24.736	22.362	19.812	7.042	5.892	5.009	4.107	3.565
14	31.319	29.141	26.119	23.685	21.064	7.790	6.571	5.629	4.660	4.075
15	32.801	30.578	27.488	24.996	22.307	8.547	7.261	6.262	5.229	4.601
16	34.267	32.000	28.845	26.296	23.542	9.312	7.962	6.908	5.812	5.142
17	35.718	33.409	30.191	27.587	24.769	10.085	8.672	7.564	6.408	5.697
18	37.156	34.805	31.526	28.869	25.989	10.865	9.390	8.231	7.015	6.265
19	38.582	36.191	32.852	30.144	27.204	11.651	10.117	8.907	7.633	6.844
20	39.997	37.566	34.170	31.410	28.412	12.443	10.851	9.591	8.260	7.434
21	41.401	38.932	35.479	32.671	29.615	13.240	11.591	10.283	8.897	8.034
22	42.796	40.289	36.781	33.924	30.813	14.041	12.338	10.982	9.542	8.643
23	44.181	41.638	38.076	35.172	32.007	14.848	13.091	11.689	10.196	9.260
24	45.559	42.980	39.364	36.415	33.196	15.659	13.848	12.401	10.856	9.886
25	46.928	44.314	40.646	37.652	34.382	16.473	14.611	13.120	11.524	10.520
26	48.290	45.642	41.923	38.885	35.563	17.292	15.379	13.844	12.198	11.160
27	49.645	46.963	43.195	40.113	36.741	18.114	16.151	14.573	12.879	11.808
28	50.993	48.278	44.461	41.337	37.916	18.939	16.928	15.308	13.565	12.461
29	52.336	49.588	45.722	42.557	39.087	19.768	17.708	16.047	14.256	13.121
30	53.672	50.892	46.979	43.773	40.256	20.599	18.493	16.791	14.953	13.787
31	55.003	52.191	48.232	44.985	41.422	21.434	19.281	17.539	15.655	14.458
32	56.328	53.486	49.480	46.194	42.585	22.271	20.072	18.291	16.362	15.134
33	57.648	54.776	50.725	47.400	43.745	23.110	20.867	19.047	17.074	15.815
34	58.964	56.061	51.966	48.602	44.903	23.952	21.664	19.806	17.789	16.501
35	60.275	57.342	53.203	49.802	46.059	24.797	22.465	20.569	18.509	17.192
36	61.581	58.619	54.437	50.998	47.212	25.643	23.269	21.336	19.233	17.887
37	62.883	59.893	55.668	52.192	48.363	26.492	24.075	22.106	19.960	18.586
38	64.181	61.162	56.896	53.384	49.513	27.343	24.884	22.878	20.691	19.289
39	65.476	62.428	58.120	54.572	50.660	28.196	25.695	23.654	21.426	19.996
40	66.766	63.691	59.342	55.758	51.805	29.051	26.509	24.433	22.164	20.707
45	73.166	69.957	65.410	61.656	57.505	33.350	30.612	28.366	25.901	24.311
50	79.490	76.154	71.420	67.505	63.167	37.689	34.764	32.357	29.707	27.991
55	85.749	82.292	77.380	73.311	68.796	42.060	38.958	36.398	33.570	31.735
60	91.952	88.379	83.298	79.082	74.397	46.459	43.188	40.482	37.485	35.534
65	98.105	94.422	89.177	84.821	79.973	50.883	47.450	44.603	41.444	39.383
70	104.215	100.425	95.023	90.531	85.527	55.329	51.739	48.758	45.442	43.275
75	110.286	106.393	100.839	96.217	91.061	59.795	56.054	52.942	49.475	47.206
80	116.321	112.329	106.629	101.879	96.578	64.278	60.391	57.153	53.540	51.172
85	122.325	118.236	112.393	107.522	102.079	68.777	64.749	61.389	57.634	55.170
90	128.299	124.116	118.136	113.145	107.565	73.291	69.126	65.647	61.754	59.196
95	134.247	129.973	123.858	118.752	113.038	77.818	73.520	69.925	65.898	63.250
100	140.169	135.807	129.561	124.342	118.498	82.358	77.929	74.222	70.065	67.328
110	151.948	147.414	140.917	135.480	129.385	91.471	86.792	82.867	78.458	75.550
120	163.648	158.950	152.211	146.567	140.233	100.624	95.705	91.573	86.923	83.852
130	175.278	170.423	163.453	157.610	151.045	109.811	104.662	100.331	95.451	92.222
150	198.360	193.208	185.800	179.581	172.581	128.275	122.692	117.985	112.668	109.142
160	209.824	204.530	196.915	190.516	183.311	137.546	131.756	126.870	121.346	117.679
170	221.242	215.812	207.995	201.423	194.017	146.839	140.849	135.790	130.064	126.261
180	232.620	227.056	219.044	212.304	204.704	156.153	149.969	144.741	138.820	134.884
190	243.959	238.266	230.064	223.160	215.371	165.485	159.113	153.721	147.610	143.545
200	255.264	249.445	241.058	233.994	226.021	174.835	168.279	162.728	156.432	152.241