HERBERT CHITEPO SCHOOL OF LAW & BUSINESS SCIENCES

DEPARTMENT OF ECONOMICS & FINANCE

FINAL EXAMINATION

BACHELOR OF COMMERCE	PART 4 SEMESTER 2
COURSE	FINANCIAL ENGINEERING
CODE	HBF427
DATE	2024
DURATION	3 HOURS

INSTRUCTION TO CANDIDATES

1. THE PAPER COMPRISES 5 QUESTIONS.
2. YOU ARE REQUIRED TO ANSWER QUESTION 1 AND ANY OTHER THREE QUESTIONS.
3. BEGIN THE ANSWER TO EACH QUESTION ON A FRESH PAGE OF THE ANSWER BOOKLET.
4. FINANCIAL AND NON-PROGRAMMABLE SCIENTIFIC CALCULATORS ARE ALLOWED IN THE EXAMINATION.
5. CANDIDATES WILL OBTAIN CREDIT FOR SHOWING ALL WORKINGS.
6. THE STANDARD NORMAL DISTRIBUTION TABLE IS ON THE LAST PAGE.

QUESTION 1 **[40 MARKS]**

Compulsory

1.1 Let $A(0) = \$90$, $A(1) = \$100$, $S(0) = \$25$ dollars and let

$$S(1) = \begin{cases} 30 \text{ with probability p,} \\ 20 \text{ with probability } 1 - p \end{cases}$$

Given $0 < p < 1$. For a portfolio with $x = 10$ shares calculate:

1.1.1 $V(0)$, [5 Marks]

1.1.2 $V(1)$ and [5 Marks]

1.1.3 K_V [5 Marks]

1.2 Suppose that the stock price on any given day can either be 5% higher or 4% lower than on the previous day.

1.2.1 Sketch a tree representing possible stock price movements over the next three days, given that the price today is \$20. [8 Marks]

1.2.2 How many different scenarios can be distinguished? [2 Marks]

1.3 Amani stock price changes only once a month: either it goes up by 20%, or it falls by 16.7%. Its current price is \$40. Interest rate is 1% per month.

1.3.1 Compute the value of Amani stock's 2-month put option with an exercise price of \$40? [8 Marks]

1.3.2 Show how the payoffs of Amani stock's 1-month put option can be replicated by holding a portfolio of stock and bonds. [7 Marks]

QUESTION 2 **[20 MARKS]**

2.1 Define the following, as used in Financial Engineering:

2.1.1 Divisibility [4 Marks]

2.1.2 Liquidity [4 Marks]

2.1.3 Short Selling [4 Marks]

2.2 Suppose that dealer *A* in New York offers to buy British pounds a year from now at a rate $d_A = \$1.58$ to a pound, while dealer *B* in London would sell British pounds immediately at a rate $d_B = \$1.60$ to a pound. Assume dollars can be borrowed at an annual rate of 4%, and British pounds can be invested in a bank account at 6%. Show how this situation provides an opportunity for a risk-free profit without initial investment? [8 Marks]

QUESTION 3 **[20 MARKS]**

3.1 Explain the term Delta Hedging as used in financial engineering. [4 Marks]

3.2 Suppose that stock prices follow a binomial tree, the possible values of $S(2)$ being \$121, \$110 and \$100. Find u and d when $S(0) = 100$ dollars. Do the same when $S(0) = 104$ dollars. [8 Marks]

3.3 Consider a stock whose price on 1 January is \$120 and which will pay a dividend of \$1 on 1 July 2012and \$2 on 1 October 2012. The interest rate is 12%. Is there an arbitrage opportunity if on 1 January 2012 the forward price for delivery of the stock on 1 November 2012 is \$131? If so, compute the arbitrage profit. [8 Marks]

QUESTION 4 **[20 MARKS]**

4.1 Outline the advantages and disadvantages of a forward contract. [6 Marks]

4.2 The non-additivity of returns for deterministic returns is often an inconvenience. Outline measures can be put in place to reduce the impact of this problem. [4 Marks]

4.3 Suppose that the stock prices in the following three scenarios are

Scenario	S(0)	S(1)	S(2)
ω_1	100	110	120
ω_2	100	105	100
ω_3	100	90	100

Their probabilities are 0.5, 0.25 and 0.25, respectively.

4.3.1 Find the expected returns:

i. $E(K(1))$, [2 Marks]

ii. $E(K(2))$ and [2 Marks]

iii. $E(K(0, 2))$ [2 Marks]

4.3.2 Compare $1 + E(K(0, 2))$ with $[1 + E(K(1))][1 + E(K(2))]$ and explain the conclusion(s) you make. [4 Marks]

QUESTION 5 **[20 MARKS]**

The forward price of a stock paying dividend *div* at time t, where $0 < t < T$, is

$$F(0,T) = [S(0) - e^{-rt}div]e^{rT}$$

Prove that an arbitrage opportunity would arise each time this equality is not achieved. [20 Marks]

END OF EXAMINATION!

Standard Normal Distribution Table

x	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000