ROBERT MUGABE SCHOOL OF HERITAGE &EDUCATION

DEPARTMENT OF SCIENCE AND TECHNICAL EDUCATION

BACHELOR OF EDUCATION SECONDARY HONOURS PRE-SERVICE/IN-SERVICE DEGREE

PART 4 SEMESTER 1 /PART 2 SEMESTER 2

EXAMINATION QUESTION PAPER

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|---|---|---|
| MODULE CODE | : | CMTS212/CMTP212/ TDSM412 |
| NARRATION | : | ANALYSIS |
| DATE | : | 2025 |
| TIME | : | 3HOURS |

MAIN PAPER

## INSTRUCTIONS

Answer all questions in Section A and any two questions in Section B

**SECTION A:** Answer all questions (60 marks)

1 (a) Explain what is meant by saying a set F in $\mathbf{R}^n$ is:

(i) open, [3]

(ii) closed, [2]

(iii) bounded, and [3]

(iv) compact. [3]

(b) Prove that the union of any two closed sets in $\mathbf{R}^n$ is closed in $\mathbf{R}^n$ [4]

(c). Without proving, state Bolzano-Weiersterass theorem for infinite sets. [3]

2. If A, B and C are any sets, prove the De Morgan's law: $A - (B \cup C) = (A-B) \cap (A-C)$. [4]

3 (a) Using the axioms of a field, show that for $a, b \in \mathbf{R}$

(i) $a(-b) = -(ab)$, [2]

(ii) $(-a)(-b) = ab$. [2]

4 (a) State and prove the Archimedean Property [6]

(b) Hence or otherwise state the Supremum Property of **R**. [2]

(c) Prove that if $x \in \mathbf{R}$ then there is a natural number $n \in N$ such that $x < n$. [5]

5 (a) Given that $f: A \rightarrow B$ and $g: B \rightarrow C$, prove that if f and g are 1-1(one to one) functions, then $g \circ f$ is a 1-1 function. [3]

(b) Give sufficient conditions for a function to have an inverse. [2]

6 (a) Define a countable set. [3]

(b) Prove that the union of a countable collection of countable sets is countable. [5]

(c) Prove that in a field F the additive identity and the multiplicative inverse are unique. [4;4]

## SECTION B

**Answer any two questions. Each question carries 20 marks.**

7 (a) Define a Cauchy sequence in $\mathbf{R}^n$ [2]

Hence state the Cauchy Convergence Criterion [3]

(b) Define the limit of a sequence $\{x_n\}$. [2]

(c) Prove that $\{x_n\}$ can have at most one limit. [7]

(d) Prove that if $\{x_n\}$ is a convergent sequence of real numbers, then the sequence $\{x_n\}$ is bounded. [6]

8 (a) Let f (x) be a continuous function of [a,b]. Define the following:

(i) A partition P of [a,b] [2]

(ii) The Riemann upper integral of f on [a,b] [4]

(iii) The Riemann lower integral of f on [a,b] [4]

(iv) When is f Riemann Integral? [2]

(b) (i) $f(x) = \begin{cases} x^2 & \text{if } x \neq 2 \\ 0 & \text{if } x = 2 \end{cases}$

Is f(x) continuous at x =2? [2]

(ii) Let f be a function defined on (a,b) and suppose f is differentiable at $c \in (a, b)$. Prove that f is continuous at c . [6]

9 (a) Let $f(x) = |x|$

(i) Show that f is not differentiable at (0,0). [4]

(ii) What conclusion can you draw from (i)? [2]

(b) Let f be a function defined on (a, b) and suppose f is differentiable at $c \in (a, b)$. Prove that f is continuous at c. [8]

(c) Hence state and prove the Mean Value Theorem for a real continuous function f defined on the interval (a, b). [6]

**END OF EXAMINATION PAPER.**