



MUNHUMUTAPA SCHOOL OF COMMERCE

DEPARTMENT OF ECONOMICS

BACHELOR OF COMMERCE DEGREE

LEVEL 2 SEMESTER 1

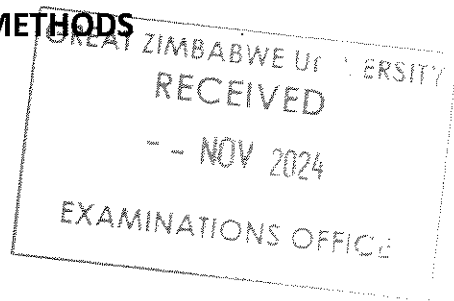
EXAMINATION QUESTION PAPER

MODULE CODE **HEF212**

MODULE NARRATION **QUANTITATIVE METHODS**

DATE **2024**

DURATION **3 HOURS**



INSTRUCTIONS TO CANDIDATES:

1. Answer any **four** (4) questions.
2. All question carry equal marks.
3. Start each answer on a fresh page.
4. Show all workings, where applicable.

QUESTION 1

(a) The demand and supply in a market are $Q^d = \sqrt{8 - P}$ and $Q^s = \sqrt{P}$

(i) Sketch both the demand & supply curves with price on the y-axis. [3 marks]

(ii) Calculate the equilibrium price and quantity [2 marks]

(ii) Solve for the consumer and producer surplus. [8 marks]

(b) If the demand and supply functions in a competitive market are;

$$Q^d = 50 - 0.2P \quad Q^s = -10 + 0.3P$$

and the rate of adjustment of price when the market is out of equilibrium is

$$\frac{dP}{dt} = 0.4(Q^d - Q^s).$$

Derive and solve the relevant difference equation to get a function for P in terms of t given that price is 100 in time period 0. Comment on the stability of this market. [12 marks]

[Total 25 marks]

QUESTION 2

(a) A person has utility function $U(x, y) = \frac{1}{3} \ln(x) + \frac{1}{4} \ln(y)$. Suppose that the price per unit of x is R3.02 and the price per unit of y is R1.76. Also the person receives R37 and spends all on the two commodities. Solve his utility maximizing problem. [9 marks]

(b) Let $R(Q)$ be the total revenue function and $C(Q)$ be the total cost function

$$R(Q) = 1000Q - 2Q^2$$

$$C(Q) = Q^3 - 62Q^2 + 1600Q + 1500$$

(i) State the profit function $\Pi(Q)$ and find the profit-maximizing output Q . [6 marks]

(ii) At what output level is the marginal revenue of the firm equal to zero. [5 marks]

(c) Solve the differential equation $\frac{dy}{dt} = 6y + 27$ if the value of y is 18 when $t = 0$. [5 marks]

[Total 25 marks]

QUESTION 3

(a) Use Cramers' Rule to solve for x_1 , x_2 and x_3 from the following system of equations.

$$x_1 - 2x_2 - x_3 = 0$$

$$-2x_1 + x_2 - x_3 = 7$$

$$x_1 - x_2 = -3$$

[13 marks]

b) Given that $A = \begin{pmatrix} 2 & 2 \\ 1 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix}$ $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

i) Find transpose of A

[3 marks]

ii) Find inverse of B

[5 marks]

iii) Find matrix C that satisfy $(A-2I)C=I$

[7 MARKS]

[Total 25 marks]

QUESTION 4

(a) Compute the rank of each of the following matrices

(i) $\begin{pmatrix} 2 & -4 & 2 \\ -1 & 2 & 1 \end{pmatrix}$

[5 marks]

(ii) $\begin{pmatrix} 1 & 6 & -7 & 3 \\ 1 & 9 & -6 & 4 \\ 1 & 3 & -8 & 4 \end{pmatrix}$

[5 marks]

lii) Calculate the determinant of matrix A is given by

$$A = \begin{pmatrix} x^2 - y^2 & x + y & x \\ x - y & 1 & 1 \\ x - y & 1 & y \end{pmatrix} \quad [5 \text{ marks}]$$

- c) A person has utility function $U(x, y) = \frac{1}{3} \ln(x) + \frac{1}{4} \ln(y)$. Suppose that the price per unit of x is R3.02 and the price per unit of y is R1.76. Also the person receives R37 and spends all on the two commodities. Solve his utility maximizing problem. [10 marks]

[Total 25 marks]

QUESTION 5

- a) Given utility function $U = (x + 2)(y + 1)$ and price of x , y and budget (B) being ,
 $P_x = 4$ $P_y = 6$ $B = 130$
- Write down the Lagrangian function [4 marks]
 - Find the optimal level of purchase \bar{x} and \bar{y} [5 marks]
 - Is the second order sufficient condition for a maximum satisfied [4 marks]
 - Does the answer in (ii) give any comparative static information? [2 marks]
- b) Find the relative extremum of the average cost function $AC = f(Q) = Q^2 - 5Q + 8$ [5 marks]
- c) Find the relative extrema of the function $y = f(x) = x^3 - 12x^2 + 36x + 8$ [5 marks]

[Total 25 marks]

END OF EXAMINATION