GREAT ZIMBABWE UNIVERSITY

HMAT**414**

GARY MAGADZIRE SCHOOL OF AGRICULTURE AND NATURAL SCIENCE

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

BSC HONORS IN MATHEMATICS: PART4 SEMESTER 2

EXAMINATION: MAIN PAPER

HMAT 414: TOPOLOGY

DATE:

Time: 3 hours

Candidates should attempt **ANY FOUR** questions from question **A1** to **A5.**

A1. (a) Define a metric space. [2]

(b) Let (X, d) be a metric space. Prove that, every open ball is open in (X,d). [5]

(c) Let (X, d) be a metric space. If $\{x_n\}$ and $\{y_n\}$ are sequences in (X, d) such that $x_n \to x$ and $y_n \to y$, show that $d(x_{n,} y_n) \to d(x, y)$. [5]

(d) Prove that the product of two compact spaces is compact. [5]

(e) Show that a topological space is countable if and only if it is compact. [8]

A2. (a) Prove that the components of a totally disconnected space are its points. [5]

(b) Prove that a topological space is a T_1 space if and only if each point is a closed point. [5]

(c) Let X, Y *and* Z be topological spaces. If $f: X \to Y$ and $g: Y \to Z$ are continuous mapping, show that $gf : X \to Z$ is also continuous. [7]

(d) Prove that the real line and the complex plane are separable. [8]

A3. (a) Let X be a topological space. If $\{X_i\}$ is a non –empty class of compact subspaces of X, show that $U_i X_i$ is also a compact subspace of X. [4]

(b) Prove that in a metric space X, the set $\emptyset$ and the full space X are open sets. [5]

(c) Let $X = (X, d)$ be a compact metric space. Prove that;

(i) X is separable, [4]
(ii) X is second countable. [5]

(d) Prove that the product of two connected spaces is connected. [7]

A4. (a) Prove that if a topological space is metrizable, then it is metrizable in an infinite number of different ways. [5]

(b) Prove that every compact T_2 space is normal. [10]

(c) State and prove Lebesgue's Covering Lemma. [10]

A5. (a) Show that a closed set is not dense if and only if its complement is everywhere dense. [5]

(b) Give four examples of countable dense sets. [5]

(c) Prove that every compact subspace of a Hausdorff space is closed. [7]

(d) State and explain in detail four separation axioms as according to Alexandroff and Hopf. [8]