GREAT ZIMBABWE UNIVERSITY

HMAT222/CMTS121

GARRY MAGADZIRE SCHOOL OF AGRIC AND NATURAL SCIENCES

ABSTRACT ALGEBRA 1

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCES

BSc Honours In Mathematics: PART 2 SEMESTER 2

Final Exam November 2024
Time : 3 hours

Candidates should attempt **ALL** questions in **SECTION A** and any **THREE** in **SECTION B**.

Instruments and Materials

- Calculator

SECTION A

Answer **ALL** questions in this section - (40 MARKS)

A1. Let $A = \{1; 2; 3; 4\}$ and $B = \{5; 6\}$. Evaluate,

(a) $A \times B$ [3]

(b) $B \times B$ [3]

A2. Let A, B and C be sets $B \neq \phi$. Given that $A \times B \subseteq C \times C$. Prove that $A \subseteq C$. [5]

A3. (a) Let $A = \{1; 2; 3\}, B = \{3; 4\}, C = \{1; 5; 7\}$. Evaluate $\Im(A; B; C)$, the set of all possible sets A, B and C. [4]

(b) Let $X = \{p; q; r\}$. Find $\Im(X)$. [5]

A4. (a) Let $A = \{1; 2; 3\}, B = \{a; b; c\}$ and $C = \{1; 2; 3; 4; 5\}$. Describe f in the following statements using injective, surjective and state which of them is a bijection.

(i) Define $f : A \to C$ by $f(1) = 1; f(2) = 2; f(3) = 3$. [3]

(ii) Define $f : A \to B$ by $f(1) = a; f(2) = b; f(3) = c$ [3]

(b) Let a and b be fixed real numbers, $a \neq 0$. Define $\theta : \mathbb{R} \to \mathbb{R}$ by $\theta(x) = ax + b \forall x \in \mathbb{R}$. Show that θ is a bijection mapping. [6]

A5. Describe in words the sets $A = \{x \in \mathbb{Z} | x \geq 2\}$ and $B = \{x \in \mathbb{Z} | x \leq 5\}$. Show that $A \cap B$ is finite and $A \cup B = \mathbb{Z}$. [8]

SECTION B

Answer any **THREE** questions- (60- MARKS)

B6. (a) Let f, g be mappings from $\mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2, g(x) = 2x - 1$. Find

(i) gf and fg. [4]

(ii) the values of x for which

$$gf(x) = fg(x)$$

[6]

(b) Let $f : A \rightarrow B$ and $g : B \rightarrow A$ be mappings such that $g\emptyset f = f\o g = I$ where I is the identity mapping on A. Prove that f is bijective and g is its inverse. [10]

B7. Let m be a fixed integer and $a, b, c, d \in \mathbb{Z}$ such that $A \equiv b \bmod m$ and $c \equiv d \bmod m$. Prove that

(a) $(a + c) \equiv (b + d \bmod m$. [7]

(b) $(a - c) \equiv (b - d \bmod m$. [7]

(c) $ac \equiv bd \bmod m$. [6]

B8. (a) Define a group. [3]

(b) Show that the following are groups

(i) $G = \{1; -1; i; -i\}$ where $G \in (\mathbb{C}^*; \times)$. [8]

(ii) $(\mathbb{Z}_6, +)$, where $\mathbb{Z}_6$ denotes the set of integers modulo 6. [7]

(c) Define a abelian group. [2]

B9. (a) Define a subgroup. [3]

(b) Consider $G = \{1; -1; i; -i\}$ where $G \in (C^*; \times)$. Find all the possible subgroups of G. [6]

(c) Let $G = \{\mathbb{R}/\{0\}; \times\}$ and $H = \{2^{nd} : n \in \mathbb{Z}\}$. Prove that $H \leq G$. [6]

(d) Let H and K be subgroups of G. Then $H \cap K$ is also a subgroup of G. [5]

B10. (a) Define a ring R. [3]

(b) Let R be a ring, e_1, the left identity of R and e_2 the right identity of R. Show that

$$e_1 = e_2$$

[5]

(c) Define the characteristic of a given ring R. [5]

(d) Consider $Z_5 = \{0; 1; 2; 3; 4\}$. Evaluate the characteristic of Z_5 [5]

(e) Define a subring. [2]

END OF QUESTION PAPER