GREAT ZIMBABWE UNIVERSITY

HMAT 4110

NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

BSc Honours in Mathematics: Part 4 Semester 1

EXAMINATION: BANK2

HMAT 4110: SET THEORY AND LOGIC

DATE: June 2024

Time: 3 hours

Candidates should attempt ANY FOUR questions. Each question carries 25 marks.

A1. (a) Prove that if A $\leq$ B and B $\leq$ A, then A$\approx$B. [8]

(b) Prove that a subset of countable set is countable. [8]

(c) State Zorn's lemma. [2]

(d) State and prove Completeness Theorem. [7]

A2. (a) Prove that if n is a finite cardinal, then $n<\aleph_0$ [5]

(b) Show that $\aleph x \aleph$ is denumerable. [10]

(c) (i) Prove that between any two distinct rational numbers there is another rational number. [5]

(ii) Prove the lemma which says if x is a Cauchy Sequence of rational numbers, then $\exists$ a positive rational number δ such that for every n, $|x_n| < \delta$ [5]

A3. (a) Prove that any set of ordinals is well-ordered. [7]

(b) Prove that the Axiom of Choice is equivalent to Hausdorff's maximal principle. [8]

(c) State and prove Cauchy Convergence principle. [10]

A4. (a) Prove that a non-empty set of real numbers which has an upper bound has a least upper bound. [10]

(b) Prove that $\models$A and $\models$ A $\rightarrow$ B, then $\models$ B. [5]

(c) (i) State the definition of a Boolean Algebra. [3]

(ii) Prove the necessary and sufficient conditions that a Boolean algebra be isormorphic to the algebra of all subsets of some set and that B be complete and atomic. [7]

A5. (a) State and prove the Soundness Theorem. [7]

(b) State and prove the Decidability Theorem of Logic. [8]

(c) Prove that $p \to p$ [10]

END OF QUESTION PAPER