GREAT ZIMBABWE UNIVERSITY

AIWE 222

GARY MAGADZIRE SCHOOL OF AGRICULTURE

DEPARTMENT OF SOIL AND PLANT SCIENCES

BSc IRRIGATION AND WATER ENGINEERING: LEVEL 4 SEMESTER 1

EXAMINATION: MAIN PAPER

AIWE 222: ENGINEERING MATHEMATICS IV

DATE:NOVEMBER 2024

Time : 3 hours

Candidates should attempt **ALL** questions in Section A and **ANY TWO** questions from Section B

SECTION A

A1. Let $f(x, y) = e^{xy}\sin(x + y)$.

(a) In what direction, starting at $(0, \frac{\pi}{2})$, is f changing the fastest? [5]

(b) In what directions starting at $(0, \frac{\pi}{2})$ is f changing at 50% of its maximum rate? [7]

(c) Let $c(t)$ be a flow line of $F = \nabla f$ with $c(0) = (0, \frac{\pi}{2})$. Calculate $\frac{d}{dt}\left[f(c(t))\right]_{t=0}$. [3]

A2. Let $f : R^3 \to R^3$ be a given mapping and write

$$f(x,y,z) = (u(x,y,z), v(x,y,z), w(x,y,z)).$$

Let $g : R^3 \to R^3$ be defined by

$$g(u,v,w) = (uv, u+w, w+v) \text{ and let } h = g \circ f.$$

(a) Write a formula for the derivative matrix $\mathbf{D}h$. [4]

(b) Show that $\mathbf{D}h$ cannot have rank 3 at any point (x,y,z). [3]

(c) Show that $\mathbf{D}h$ has an eigenvalue zero at every (x,y,z). [2]

A3. Extremize $f(x,y,z) = x$ subject to the constraints $x^2+y^2+z^2 = 1$ and $x+y+z = 1$. [9]

A4. (a) Evaluate

$$\int\int\int_D exp[(x^2+y^2+z^2)^{\frac{3}{2}}]dxdydz$$

where D is the region defined by $1 \leq x^2+y^2+z^2 \leq 2$ and $z \geq 0$. [4]

(b) Sketch or describe the region of integration for

$$\int_0^1 \int_0^x \int_0^y f(x,y,z)dzdydx,$$

and interchange the order to $dydxdz$. [3]

SECTION B

Answer ANY TWO questions [60 MARKS]

B5. Let $\mathbf{G}(x, y) = (xe^{x^2+y^2} + 2xy)\mathbf{i} + (ye^{x^2+y^2} + x2)\mathbf{j}$.

(a) Show that $\mathbf{G} = \nabla\mathbf{f}$ for some $\mathbf{f}$; find such an $\mathbf{f}$. [10]

(b) Use (a) to show that the line integral of $\mathbf{G}$ around the edge of the triangle with vertices $(0, 0), (0, 1), (1, 0)$ is zero. [10]

(c) State Green's theorem for the triangle in (b) and a vector field $\mathbf{F}$ and verify it for the vector field $\mathbf{G}$ above. [10]

B6. Let $\mathbf{W}$ be the three dimensional region under the graph of $\mathbf{f}(x, y) = exp(x^2 + y^2)$ and over the region in the plane defined by $1 \leq x^2 + y^2 \leq 2$.

(a) Find the volume of $\mathbf{W}$. [13]

(b) Find the flux of the vector field $\mathbf{F} = (2x - xy)\mathbf{i} - y\mathbf{j} + yz\mathbf{k}$ out of the region $\mathbf{W}$. [17]

B7. Let $\mathbf{C}$ be the curve $x^2 + y^2 = 1$ lying in the plane $z = 1$. Let $\mathbf{F} = (z - y)+y\mathbf{j}$.

(a) Calculate $\nabla \times \mathbf{F}$. [6]

(b) Calculate $\int_C \mathbf{F}.ds$ using a parametrization of C and a chosen orientation for C. [8]

(c) Write $C = \partial S$ for a suitably chosen surface S and, applying Stokes's theorem, verify your answer in (b). [8]

(d) Consider the sphere with radius $\sqrt{2}$ and center the origin. Let S' be the part of the sphere that is above the curve (i.e., lies in the region $z \geq 1$), and has C as boundary. Evaluate the surface integral of $\nabla \times \mathbf{F}$ over S'. Specify the orientation you are using for S'. [8]

END OF QUESTION PAPER