

GREAT ZIMBABWE UNIVERSITY

AIWE 222

GARY MAGADZIRE SCHOOL OF AGRICULTURE

DEPARTMENT OF SOIL AND PLANT SCIENCES

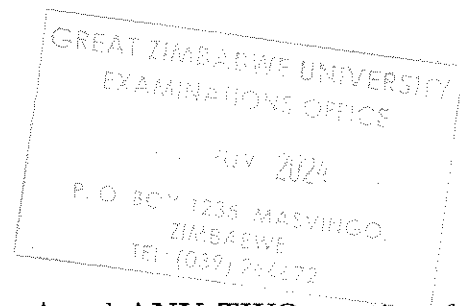
BSc IRRIGATION AND WATER ENGINEERING: LEVEL 4 SEMESTER 1

EXAMINATION: MAIN PAPER

AIWE 222: ENGINEERING MATHEMATICS IV

DATE:NOVEMBER 2024

Time : 3 hours



Candidates should attempt **ALL** questions in Section A and **ANY TWO** questions from Section B

SECTION A

A1. Let $f(x, y) = e^{xy} \sin(x + y)$.

- (a) In what direction, starting at $(0, \frac{\pi}{2})$, is f changing the fastest? [5]
- (b) In what directions starting at $(0, \frac{\pi}{2})$ is f changing at 50% of its maximum rate? [7]
- (c) Let $c(t)$ be a flow line of $F = \nabla f$ with $c(0) = (0, \frac{\pi}{2})$. Calculate $\frac{d}{dt} [f(c(t))]_{t=0}$. [3]

A2. Let $f : R^3 \rightarrow R^3$ be a given mapping and write

$$f(x, y, z) = (u(x, y, z), v(x, y, z), w(x, y, z)).$$

Let $g : R^3 \rightarrow R^3$ be defined by

$$g(u, v, w) = (uv, u + w, w + v) \text{ and let } h = g \circ f.$$

- (a) Write a formula for the derivative matrix Dh . [4]
 (b) Show that Dh cannot have rank 3 at any point (x, y, z) . [3]
 (c) Show that Dh has an eigenvalue zero at every (x, y, z) . [2]

A3. Extremize $f(x, y, z) = x$ subject to the constraints $x^2 + y^2 + z^2 = 1$ and $x + y + z = 1$. [9]

A4. (a) Evaluate

$$\int \int \int_D \exp[(x^2 + y^2 + z^2)^{\frac{3}{2}}] dx dy dz$$

where D is the region defined by $1 \leq x^2 + y^2 + z^2 \leq 2$ and $z \geq 0$. [4]

(b) Sketch or describe the region of integration for

$$\int_0^1 \int_0^x \int_0^y f(x, y, z) dz dy dx,$$

and interchange the order to $dy dx dz$. [3]

SECTION B

Answer ANY TWO questions [60 MARKS]

B5. Let $\mathbf{G}(x, y) = (xe^{x^2+y^2} + 2xy)\mathbf{i} + (ye^{x^2+y^2} + x^2)\mathbf{j}$.

- (a) Show that $\mathbf{G} = \nabla f$ for some f ; find such an f . [10]
- (b) Use (a) to show that the line integral of \mathbf{G} around the edge of the triangle with vertices $(0, 0)$, $(0, 1)$, $(1, 0)$ is zero. [10]
- (c) State Green's theorem for the triangle in (b) and a vector field \mathbf{F} and verify it for the vector field \mathbf{G} above. [10]

B6. Let \mathbf{W} be the three dimensional region under the graph of $f(x, y) = \exp(x^2 + y^2)$ and over the region in the plane defined by $1 \leq x^2 + y^2 \leq 2$.

- (a) Find the volume of \mathbf{W} . [13]
- (b) Find the flux of the vector field $\mathbf{F} = (2x - xy)\mathbf{i} - y\mathbf{j} + yz\mathbf{k}$ out of the region \mathbf{W} . [17]

B7. Let C be the curve $x^2 + y^2 = 1$ lying in the plane $z = 1$. Let $\mathbf{F} = (z - y)\mathbf{i} + y\mathbf{j}$.

- (a) Calculate $\nabla \times \mathbf{F}$. [6]
- (b) Calculate $\int_C \mathbf{F} \cdot d\mathbf{s}$ using a parametrization of C and a chosen orientation for C . [8]
- (c) Write $C = \partial S$ for a suitably chosen surface S and, applying Stokes's theorem, verify your answer in (b). [8]
- (d) Consider the sphere with radius $\sqrt{2}$ and center the origin. Let S' be the part of the sphere that is above the curve (i.e., lies in the region $z \geq 1$), and has C as boundary. Evaluate the surface integral of $\nabla \times \mathbf{F}$ over S' . Specify the orientation you are using for S' . [8]

END OF QUESTION PAPER